THE

STRAINS UPON BRIDGE GIRDERS AND ROOF TRUSSES.

THE
STRAINS UPON BRIDGE GIRDERS AND ROOF TRUSSES.

INCLUDING

THE WARREN, LATTICE, TRELLIS, BOWSTRING, AND OTHER FORMS OF GIRDERS, THE CURVED ROOF, AND SIMPLE AND COMPOUND TRUSSES.

BY

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With Sixty-four Illustrations on Wood,
DRAWN AND WORKED OUT TO SCALE.

Nihil sine labore.

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PREFACE.

A SERIES of articles contributed to the scientific journal 'The Engineer' in the years 1870–71, constituted the nucleus of the present treatise. They were written at such periods of leisure, as the Author could spare from the time devoted to his professional duties. The method of determining the strains upon a structure, by the aid of graphic diagrams, is at once accurate and elegant, and recommends itself especially to the engineering student and beginner, by its simplicity and conclusiveness. The particular method adopted by the Author, is that, in which, each successive strain is shown in diagrams, so that the manner in which, the total strain upon any member of a structure is produced, may be thoroughly understood. The manner in which the strains are transferred from one member to another, can be thus traced throughout the whole design, which is not the case with mathematical calculation, the chief object of which is to furnish the total results, and the total results only, of the action of the load upon the different parts of the girder or truss. Moreover, the process of analytical calculation, when applied in detail, even to those simpler examples of construction, to which it is alone applicable, becomes extremely tedious and laborious. As a check upon the accuracy of the general

results arrived at by the graphic method, it can be employed with advantage, and is introduced for that purpose in the present volume.

It cannot be denied, that a method of arriving at any desired result in the field of science, which appeals to the senses as well as to the mind, must be more congenial to the comparatively untrained intellect, than that which calls into play the mental faculties only. In the former case we perceive, and therefore understand; in the latter, whether we understand or not, we perceive nothing. Many persons, in fact the majority, have either a natural distaste for intricate mathematical investigations, or, from imperfect education or other cause, are unable to follow them with any degree of satisfaction to themselves. The method of graphic diagrams offers the facilities denied by the other and more complicated principle, and thus enables the same goal to be reached by a different road. The result is the same, although the means, employed to arrive at that result, are different. When a girder or roof truss belongs to a certain type of design, it becomes impracticable to determine the strains by mathematical calculation. As diagrams must, therefore, be resorted to in those instances, in which the designs are of a complicated character, it is absolutely necessary, that the manner of applying them to the simple cases, should be thoroughly understood. This necessitates a study of the whole subject, which, the Author trusts, the present treatise will render free from difficulty.

LONDON, 1ST MARCH, 1873.

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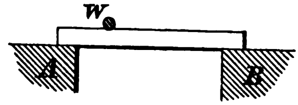
THE STRAINS UPON BRIDGE GIRDERS AND ROOF TRUSSES.

CHAPTER I.

INTRODUCTORY.

To ensure the greatest amount of strength with the least quantity of material, is the problem to be solved by everyone engaged in the art of construction. The solution of this problem, which is imperative upon every engineer, demands a sound and accurate knowledge of the strength of the materials to be employed, and an intimate acquaintance with the best forms in which they may be disposed. The most general cause of all strains and pressures with which the engineer has to deal, is gravity, or weight. If a beam be imagined absolutely without weight, and placed as represented in Fig. 1, it would be altogether free from strain or pressure in any part. But if a weight W be placed upon it, strains are developed throughout its entire length, which are transmitted to the two supports A and B,

FIG. 1.

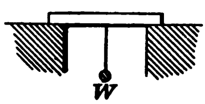


and the case at once becomes a subject for calculation. The effect of the application therefore, of any external force, weight, or pressure upon a body, is to produce

strain in that body. By the word strain, may be understood the motion, or tendency to motion, which occurs among the molecules or atoms of a body when subjected to an external force. Thus the breaking strain of a beam or bar, is the strain caused by the application of an external force sufficiently great to fracture or break it, and so produce the greatest possible motion among its particles. The motion of the relative particles may be either towards one another, as occurs when a body is compressed, or in the opposite direction, as when it is stretched. These points will become clearer, as the various strains to which bodies are liable are treated of.

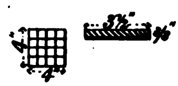
A strain of tension, or a tensile strain, is the strain produced in a body by what is commonly called pulling it, and is the opposite of a compressive strain, which results from pushing it. Familiar examples of tensile strain are to be found in the cables of ships, when either weighing, or riding at anchor; in the chains and ropes of cranes, when engaged either in raising or lowering of weights; in the coupling bars of railway carriages, the rods and chains of suspension bridges, and in numerous instances which will present themselves to our readers. A simple kind of tensile strain is represented in Fig. 2, where a weight W is suspended, by means of a rod of wrought iron, from a beam overhead, and the practical point is to find, when the weight is given, what the size of the rod should be to support it in safety. Before, however, this can be ascertained, the actual breaking weights of wrought-iron rods and bars must be determined, or, more generally, that of wrought iron of any form. Taking the average result of numerous experiments,

FIG. 2.



the breaking tensile strain of good wrought iron, such as may be obtained without inflicting a stringent specification, or imposing unfair responsibility on the manufacturers, may be taken at 22 tons per square inch. It may be remarked here that the experiments undertaken upon bars, plates, and other forms of wrought iron, for the purpose of arriving at the breaking strain, were made upon specimens of different sizes, so that, in order to compare them, they were all reduced to the strain upon one inch of sectional area. The following explanation will render this clear, as well as the meaning of the term "sectional area." In Fig. 3 is shown a section of a square bar of wrought iron 4″ × 4″, of which the sectional area is 16 inches, as seen by the divisions in the Fig. If this bar be placed in the situation represented in Fig. 2, and the weight W be found to have torn or pulled it asunder, when it was equal to 352 tons, then to find what the breaking weight per inch of sectional area was, or of one of the small divisions in the Fig., divide the total weight by the number of divisions of inches of sectional area, and the calculation is $\frac{352}{16} = 22$ tons. In future calculations, with respect to the tensile strain of wrought iron, 22 tons per inch of sectional area will be taken to represent the breaking strain. Some experiments have given a much greater tensile strength to the same material, but it is safer, and much more correct, in nine cases out of ten, to provide not for the superior descriptions of iron, but for the ordinary.

FIG. 3.



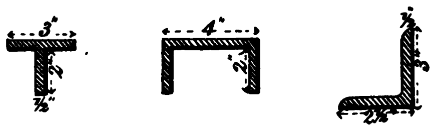
Having settled upon the standard to be adopted, one of the simplest problems presenting itself may now be

solved, but nevertheless one which is constantly occurring. Given a bar of certain size, what weight suspended from it will break it? The rule is, "Multiply the number of inches in the sectional area of the bar by the standard number, and the product will be the breaking strain or weight in tons." As an example, take the bar shown in Fig. 3, having a breadth of $3\frac{1}{2}''$ and a thickness of $\frac{3}{8}''$. Its sectional area will consequently be obtained by multiplying these two dimensions together, and will equal $3\frac{1}{2}'' \times \frac{3}{8}''$, or, expressed in inches and decimals, $3 \cdot 5 \times 0 \cdot 0375 = 1 \cdot 31$, which is the number of inches in the sectional area, or, briefly, the sectional area of the bar. Multiplying this sectional area by the standard number, the product is $28 \cdot 82$ tons, or, in round numbers, the breaking weight of the bar is 29 tons. Putting the calculation in an algebraical form, let C be the constant or standard number, B the breadth of the bar, D its thickness, N the number of inches in the sectional area, and W the breaking weight in tons, then $W = N + C = B \times D \times C$. Again, let an iron rod be $2\frac{1}{4}''$ in diameter, what will be its breaking weight? The same rule applies as in the case of a rectangular bar. The sectional area is equal to the square of the diameter multiplied by the number $0 \cdot 7854$, and the calculation will stand $2 \cdot 25 \times 2 \cdot 25 \times 0 \cdot 7854 \times 22 = 87 \cdot 473$ tons. With respect to a tensile strain, the form of the bar has no influence on its strength, the latter being in the direct proportion of the sectional area, which is not the case with a strain of compression.

The strength, moreover, of a bar exposed to a tensile strain is in certain positions independent of the length. The above calculation may be written as follows: Putting R for the radius of the rod, and D for its diameter, we

have as before $W = N \times C = \pi \dfrac{D^2 \times C}{4}$. Let us consider the case of an angle iron $3'' \times 2\frac{1}{2}'' \times \frac{1}{2}''$, represented in Fig. 4. Its sectional area will be equal to the sum of the two sides, less the thickness, multiplied by the thickness, thus $(3 + 2\cdot5 - 0\cdot5) \times 0\cdot5 = 2\cdot5$ square inches, and this multiplied by the constant equals a breaking weight of 55 tons. Similarly the breaking weights of the T and channel forms, shown in Fig. 4, will be found upon calculation to be 55 and 88 tons respectively.

FIG. 4.



The three sections in Fig. 4 are better adapted for undergoing a compressive than a tensile strain, but they have been given here for the sake of illustrating the rule; and, moreover, in large lattice bridges they are frequently used, the angle iron especially, in situations where, although their principal strain is of a tensile character, yet they are also, under certain conditions of loading, subjected to a small compressive one; besides, they can be always advantageously employed where lateral stiffness is required, whatever may be the nature of the strain they may have to undergo in the direction of their length.

The proportion which the actual weight put upon either a single bar or a compound structure in practice, or the working load, as it is called, should bear to the breaking weight, has long been an undecided question. Without inquiring into the differences of opinion expressed by various engineers and mathematicians upon the subject,

it is sufficient to state that the limits are three and ten; that is, one extreme is that the breaking weight should be three times and the other ten times the working load. Engineers are still not unanimous on this point; but it is generally considered that the safe working load may be the one-fourth of that which would actually fracture the material. Although there are objections to the regulations of the Board of Trade, with reference to the strength of all structures coming under their inspection, yet it is very doubtful whether, all things considered, a safer rule, so far as the public are concerned, could be laid down than that which provides that the tensile strain upon wrought iron shall not exceed 5 tons per square inch. Instead, therefore, of taking the safe working load of wrought iron, when subject to a tensile strain, at one-fourth of that of the breaking weight, that is at 5·5 tons per inch of sectional area, it will be considered equal to 5 tons. In order to obtain the safe load to be put on a bar of given dimensions, the rule already given will require a slight alteration, and will run thus: "Multiply the sectional area of the bar in inches by the standard number 5, and the product will be the safe working strain in tons." Given a bar $3\frac{1}{4}'' \times \frac{5}{8}''$, what will be its safe working tensile strain in tons? $3\frac{1}{4} \times \frac{5}{8} \times 5 =$ in round numbers $10\frac{1}{4}$ tons. In the preceding examples the dimensions of the bar have been assumed to be known, and the breaking or safe working load the unknown quantity to be determined, but it is quite as frequent an occurrence for the weights to be given, and the other dimensions of the bar to be required. Without regarding what the shape of the bar may be, let it be required to determine what should be the sectional area of a bar which will support safely a weight of 30 tons.

Rule: "Divide the weight in tons by the number 5, and the quotient is the sectional area in inches." A sectional area therefore of 6 inches will safely carry a weight of 30 tons. To find the diameter of a round bar which is to carry a certain safe load: "Divide the weight in tons by the number 5; divide the quotient by the number 0·7854, and the square root of the number will be the diameter required." As an example: What will be the diameter of a rod capable of safely supporting 28 tons? Answer: 2·25″. A rather simpler, though not quite so accurate a rule as the foregoing, is the following: "Take the square root of the weight in tons, and divide by two." By this rule the diameter would be equal to 2·236″, the difference being very trifling, and the labour of calculation very considerably reduced.

The converse of the proposition may be worked out in a manner equally simple. Let it be required to find what weight would be safely supported by a rod of a given diameter. Rule: "Square the diameter in inches, and multiply the result by 4; the product will be the safe working load in tons." What weight would a rod 3 inches in diameter safely support? Answer: 36 tons.

Rectangular bars may be regarded as those in which the breadth and the thickness may be in any relative proportion to one another. When these two dimensions are equal, the bar of course becomes square in section. There are practical limits to the ratio between the breadth and thickness of bars that should not be passed. As a rule a bar should not be less than $\frac{3}{8}$ inch in thickness, nor do they usually exceed 1 inch. Whenever it is necessary to employ a bar of excessive scantling, it is preferable to use a couple, not only because it is difficult to roll iron of a uniform and homogeneous texture when it surpasses

certain dimensions, but also the strain is more evenly distributed over the material, when a couple of smaller bars are used in the place of one very large one. It is besides a fact well established by experiments, that small bars and rods are *proportionally* stronger than large ones. To find what should be the dimensions of a rectangular bar to support a given weight, it is evident that as the requisite sectional area will depend upon the product of those two dimensions, the breadth into the thickness, one of them must be assumed before the other can be found. If the breadth be given, then to find the depth or thickness we use the following rule:—"Divide the load in tons by five times the given breadth in inches, and the quotient will be the required thickness in inches." Should the thickness be given the rule will stand. "Divide the load in tons by five times the given thickness in inches, and the quotient will be the required breadth in inches." As an example: What should be the thickness of a rectangular bar 3 inches broad to support a tensile strain, or weight, of 12 tons? Answer: $\frac{4}{5}$ inch or $\frac{13}{16}$ inch. But as bars increase usually by $\frac{1}{8}$ inch in thickness, that dimension must always be expressed in multiples of that subdivision, so that practically the answer to the above would be $\frac{7}{8}$ inch, and the bar would be 3 inches by $\frac{7}{8}$ inch. It will be always safer to add a $\frac{1}{16}$ inch to the bar than to subtract it, or, in other words, to make it a little stronger than necessary, rather than a little weaker. When the bar is square the dimension required is the length of one of its sides, which may be found as follows: "Divide the weight in tons by 5, extract the square root of the quotient, and the result will be the length of the side in inches." Example: What should be the length of the side of a square bar to support 20 tons?

Answer: 2 inches. Similarly with bars of any section whatever, where it may not be possible to determine the dimensions so readily, let the sectional area be first obtained, and then the dimensions necessary to make up that amount of metal may be determined by one or two trials. Obviously, in the case of channel iron, we must find what is the total quantity of material or sectional area required, and then proceed to adapt the proportions so as to constitute that area. In all instances of calculation respecting sections of iron, different in form from the round and rectangular bar, the designer should make himself acquainted with the *ordinary* sections and scantlings to be had in the market. It never pays to order special forms of iron, unless the job be a very large one —such, for instance, as the new roof over the Midland Terminus at St. Pancras. In instances of so extensive a character, it is worth the manufacturers' while to construct especial rolls to turn out the large quantities demanded.

CHAPTER II.

ELONGATION, ELASTICITY.

It has been already stated that the effect of the application of any force to a body, is to induce motion among its component particles, or, in other words, to alter the original shape of the body. The effect of the strain we are at present considering, is manifestly to alter the form of the body in the direction of its length. A weight suspended from a rod tends to lengthen it or stretch it. While it is impossible to ascertain the smallest weight which will stretch a rod, or under what strain it commences to be lengthened, it is quite possible to determine what is the greatest weight that may be brought upon it without damaging its elasticity, or the greatest amount of extension the rod should be allowed to undergo with safety. The elasticity of a body is the peculiar property it possesses of returning to its original form after the application of a force. This is a property shared in a greater or less degree by all bodies. Whether a body, after being acted upon by an external force, will return absolutely to the same form it had before the application of the force, depends upon three conditions, namely, the amount of the force, the degree of elasticity possessed by the body, and the duration of the time it is allowed to be subjected to the force. If we imagine a weight of 10 tons to stretch a rod to a certain extent, it is reasonable that a weight of 20 tons would stretch it still more, if not to double the former extent. This is the general law of elasticity,

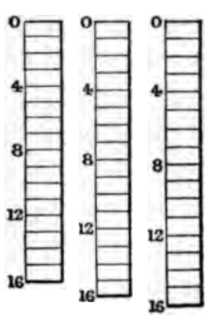
which is usually known as Hooke's law, and was propounded by him as "*ut tensio sic vis*," signifying that the extension is proportional to the force applied. There is no necessity to question the abstract truth of this law with respect to different bodies; it is sufficient to know that within certain limits it is practically true for all purposes of construction. The limits alluded to are those where an excessive weight is applied and the increase of length is also excessive, in which case the law no longer holds, and the extension augments in an irregular and dangerous manner. It is to guard against any violation of the law, and of danger of fracture to the rod, that the safe working load has been chosen so as to fall well within the limits of the elasticity of the material.

Under ordinary circumstances an iron bar, after supporting a considerable load, will not return to its original length, but will undergo a permanent alteration in that direction. This permanent increase of length is termed the "set," and its amount depends upon the force applied and the nature of the material. When a bar is subjected simply to its safe working load, there is no appreciable set, but, as it becomes necessary to test bars in order to ascertain the quality and strength of the iron, a heavy strain must be applied, and the "set" is to some extent an indication of the character of the material. It is easy to see that some care is demanded in not overdoing the experiment, for if the weight be too great, and the "set" of a corresponding magnitude, the elasticity of the iron is injured, and the bar rendered useless. There are some peculiarities attending the "set" of iron deserving of attention. In the first place, it is not produced instantaneously, but some time is required for it to acquire its full amount due to a given weight or strain. When once this

has taken place and the weight been removed, the second application of it, or of any smaller weight, produces no further "set" or permanent elongation in the material. Should a considerably greater weight be applied, then the bar will undergo another elongation or set, due to the greater strain upon it. It appears, as if a certain duration of time were necessary, to enable the material to adapt itself to the particular circumstances of each case, for if a heavy weight be suddenly and rapidly applied to a bar, it will break, or rather snap at once, without undergoing any elongation of its length; the strain is induced so quickly that the elastic force has no time, to use a common phrase, to exert itself.

There is evidently, therefore, a particular position or distance of the particles of a bar among themselves, the best suited to resist different degrees of strain. In Fig. 5 let the same bar be represented under three different conditions. Assume at first that it has just come from the rolls, and has not been subjected to any appreciable strain, and let the horizontal lines in the diagram represent the supposed distance apart of its component particles. Secondly, let us imagine it strained in tension by a weight of 20 tons; it will suffer an elongation, and the distance between the particles will be increased, which is shown in the figure by the horizontal lines being placed farther apart. This distance accordingly is the best adapted for resisting a strain of the given amount, and will therefore not be altered by the renewal or reapplication of it, no matter how often the operation may be repeated. So soon as a

FIG. 5.



strain greater than that which has already been applied, is brought upon the bar, it takes another "set," and assumes a new relative arrangement of particles, shown in the cut, and this process of self-adaptation to resist the strain goes on, until the elastic powers of the material are overcome, and the bar breaks in two. Although, for the sake of explanation, the increase of length has been represented to be considerable, in reality it is generally but very trifling, and in the majority of instances almost inappreciable. At the same time it is of importance to know how to estimate the amount of "set," and to calculate how much a bar will increase in length under the application of a given weight. It is not difficult to derive a rule for this purpose. As in former examples, a constant must first be determined, and then applied to the calculation in question. This constant is generally known as the modulus of elasticity, and is also called by some writers the coefficient of elasticity, or of elastic reaction. It is the weight in pounds that would stretch, or compress a bar, having a sectional area of one square inch, by an amount equal to its own length. In all the calculations made by the help of this coefficient, it is taken for granted that the bar has not been subjected to any previous strain, sufficiently great to have produced any permanent elongation or set. Should it have been subjected to an initial strain, it is evident, from what has been already stated, that the results would be unreliable. For wrought iron the modulus of elasticity may be taken at 24,000,000 lbs., which supposes that the strains applied are within the limits of elasticity, and consequently obey Hooke's law. If we term this the modulus of elasticity, or coefficient of elastic tension, there will be an equivalent number corresponding to the coefficient of elastic compression.

These two, for all practical purposes, may be regarded as identical, particularly as the absolute difference between the two is not of any considerable extent.

Since the modulus of elasticity is a constant, the elongation that a rod of wrought iron will undergo, upon being subjected to a given strain, will depend upon the original length of the rod, the number of square inches in its sectional area, and the amount of the straining force. The sectional area divided by the straining weight will obviously give the tension per square inch upon the rod, and the following proportion will hold :—Elongation in inches : original length in inches :: straining force per square inch of sectional area : modulus of elasticity. To obtain the elongation, the rule is, "Multiply the original length of the rod in inches by the total straining weight in pounds; divide this product by the modulus of elasticity, multiplied by the number of square inches in the rod, and the quotient will be the elongation in inches." Let L = the original length of the rod, L′ its required elongation, S the total straining weight, A the number of square inches in the sectional area of the rod, and M the modulus of elasticity; then

$$L' : L :: . \frac{S}{A} : M, \text{ and } L' = \frac{L \times S}{M \times A}.$$

M is in pounds, and the other dimensions in inches. As an example, let us take a bar 3 in. by ½ in. and 20 ft. long, strained with a weight of 20 tons—what amount of elongation will it undergo?

The calculation will stand

$$\frac{20 \times 12 \times 20 \times 2240}{1 \cdot 5 \times 24000000}$$

and the elongation in inches will be equal to $0 \cdot 298$, or

practically 0·3 of an inch. It is evident that this calculation may be abbreviated, and the number of figures reduced, which will give us a simpler rule. "Multiply the weight in tons by the original length of the bar in feet; divide by the number of square inches in the bar, multiplied by the constant 893, and the quotient will be the required elongation in inches." Should either of these rules be employed for working out the converse of the proposition, or for finding any of the other quantities, the answer will be in tons and feet. The modulus of elasticity cannot be obtained by this rule, but must be deduced from the former. The accuracy of the example given may be checked by another method. It is assumed by engineers that the weight of one ton, suspended at the end of a rod which has a sectional area of exactly one square inch, will stretch it by an amount equal to the $\frac{1}{10000}$ part of its original length, and, as has been already stated, a weight of two tons will stretch it to the $\frac{2}{10000}$, and so on within certain limits. The length of the bar selected as an example was 20 ft., or 240 in.; the total straining force was 20 tons, and the number of square inches in its sectional area 1·5. Dividing the weight by the area, the strain per square inch is 13¼ tons. Therefore the elongation in accordance with the above assumption should be equal to the $\frac{1}{10000}$ part of the original length multiplied by 13¼, that is, equal to $0 \cdot 0240 \times 13 \cdot 25$. This will give the elongation equal to 0·31, which is a sufficiently near practical approximation to the former result. The assumption of the effect of a weight of one ton upon a bar having 1 in. of sectional area, would not give a modulus of elasticity equal to what we have taken; but in addition to the fact that experimentalists differ regarding its precise value, the discrepancy is of no consequence. A

hundred tons one way or the other, would not cause an alteration in the result extending beyond the third or fourth places of decimals. In round numbers, 10,000 tons may be safely assumed as the modulus of elasticity for both the compression and tension of wrought iron, 10,356 tons being that taken by the late Professor Hodgkinson. The absolute value of the constant will vary with the quality of the iron, but it would be foreign to the purpose to investigate the details of the subject. A tensile strain, exceeding 12 tons to the square inch will injure the elasticity of wrought iron, and permanently damage its utility and strength.

CHAPTER III.

CAST IRON.

All the foregoing remarks, respecting elongation and the elasticity of iron, hold good for that material under a compressive strain. The laws governing the behaviour of all bodies under direct compression are analogous, although not quite identical. A weight placed upon a small cylinder of any material will obviously tend to compress and shorten it, and in the case of wrought iron, the compression or shortening of the cylinder, is practically proportional to the weight applied. As the compressive strength of wrought iron is inferior to its tensile resistance, and may be taken at 18 tons to the square inch, it would not be desirable to place a working strain greater than 4 tons per square inch upon it. This may to some appear too small a proportion, but there is a great difference in the character of the two strains. Even supposing that the ultimate resistance of wrought iron were the same for both compression and tension, yet the iron would be comparatively weak when exposed to the former description of strain. There is always a slight tendency to flexure, unless in very short specimens, when a material is under a compressive strain. This element of weakness is either wanting in the character of a tensile strain, or can be readily guarded against. Iron bars undergoing compression require to be secured from lateral flexure, or they no longer follow the simple law of their strength, which is directly proportional to their sectional area, and independent of their length. When this precaution is

taken, as it always should be, the crushing weight in tons of any bar or short cylinder, will be obtained by simply multiplying the sectional area in inches by the number 18, and the safe load will be the product of the sectional area in inches and the number 4. The same rules that have been given for ascertaining the ultimate and safe practical load in tension will apply to iron in compression when prevented from bending, only care must be taken to substitute the ultimate and safe working compressive resistances for those of tension.

From the very small tensile resistance possessed by cast iron, that material should never be exposed to tensile strain. Accurate experiments have determined the ultimate tensile strength of cast iron to be on an average $7\frac{1}{2}$ tons per inch of sectional area. Most erroneous conclusions have been arrived at by many eminent authorities respecting this point, some attributing to cast iron a breaking strength under tension of 20 tons. All these incorrect estimates were based upon calculation and comparison, and therefore there is no wonder that they proved false when submitted to the test of experimental fracture. It would not be prudent to subject cast iron in practice, to a tensile strain exceeding $1\frac{1}{2}$ tons per square inch. The general rules that have already been given for ascertaining the dimensions of wrought-iron rods and bars when subjected to a tensile strain, will also hold equally good for those of cast iron, care being taken to employ the proper constants, which are 7·5 tons for the breaking, and 1·5 tons for the safe, load. Mr. Hodgkinson was the first to notice, that the behaviour of cast iron under extension, differs from that presented by wrought iron. It has been already stated, that practically the extension or elongation of a wrought-iron bar, is in proportion to the weight or force tending to stretch

it. This ratio does not hold in the case of a cast-iron bar, which, instead of being elongated $\frac{1}{10000}$ part of its length for every ton of strain per square inch, is stretched to double this amount, or the $\frac{1}{5000}$ part, and the material, moreover, does not obey the law, as the magnitude of the strain is increased. Judging from the different internal structure of the two materials, one is at first inclined to consider the elongation of cast iron as somewhat paradoxical, and to be surprised that it should be double that of wrought iron for the same strain and sectional area. A little reflection will point out that this proceeds from confounding the immediate effect of the strain upon the iron, with that which it produces, when the breaking weight is reached. Thus, although cast iron extends twice the amount of wrought with a given strain, yet its ultimate tensile strain is barely a third of that of the latter material. Cast iron may be said to extend too quickly—too much at a time—and consequently its ultimate powers of tensile resistance are very soon arrived at. Wrought iron, on the contrary, is not affected so rapidly, or to the same extent, and is enabled to bear in consequence a greater strain before its resisting capabilities are overcome. Although of little value for resisting strains of tension, cast iron compensates for its weakness in that particular, by the enormous strength it possesses against those of a compressive nature. Were it not for other disadvantages connected with its use, this circumstance alone would ensure its adoption, in every instance where strains of compression were to be resisted. Its treacherous nature has, however, put almost a limit to its application to structures of any magnitude, particularly when its failure might be attended with risk to human life. Within certain limits, and when a form suitable for resisting

compression is bestowed upon it, there is no danger in employing it, and there are numerous instances where wrought iron is substituted for it, at a great sacrifice of simplicity and economy.

The crushing weight, or the ultimate resistance of cast iron to a strain of compression, may be taken at 50 tons per square inch, and the general conditions, permitting this constant to be employed in all calculations, are that the strength is directly proportional to the sectional area, and independent of the length, provided the rod, bar, or whatever example may be selected, is perfectly secured against flexure. In long pillars these conditions do not hold, owing to the tendency they have to deflect, which produces an incipient degree of weakness. Long pillars, in addition to the direct compressive strain brought upon them, are also acted upon by one of a transverse nature. In any example, therefore, of cast iron which is either too short to bend, or is secured from flexure by proper means, the crushing weight in tons is found by multiplying its sectional area in inches by 50. Moreover, since the area of a round rod is proportional to the square of its diameter, the following short practical rule will give the breaking weight in tons :—" Multiply the square of the diameter by the constant 39." This rule gives a result slightly less than the theoretical crushing weight, and consequently is a safe one to employ. If we adopt the same proportion, for the ratio of the working to the crushing load of cast iron, that we have selected in the case of wrought iron, it ought to carry safely 12½ tons per square inch. For many reasons, it would be very injudicious to place more than 8 tons upon a material so unreliable, particularly if it is likely in addition to the load, to undergo any vibration or sudden shock. Summing up results, the tensile strength

of wrought iron may be put at 22 tons per square inch, and safe working load 5 tons; the compressive strength of wrought iron at 18 tons per square inch, and safe working strain at 4 tons per same unit; the tensile strength of cast iron at 7·5 tons, and safe load at 1·5 tons per square inch; the compressive strength of cast iron at 50 tons per square inch, and safe load at 8 tons per same unit. It is not to be understood that these values represent those, that it would be absolutely dangerous to exceed, or that they are never to be departed from under any circumstances, but that they are merely sound and safe data, upon which to base calculations connected with the strength of iron. Experienced engineers may diverge a little from the beaten track, but those who may not be included within that category, will do well to confine themselves strictly within these limits. With the exception of Brunel, no authority upon the subject of the ratio that ought to be maintained, between the safe working and the ultimate strain of iron, has pronounced for a proportion exceeding one-fourth. Brunel himself considered, that an engineer might, with safety work in practice, up to one-third or two-fifths of the ultimate resisting powers of the material, but this opinion has generally been regarded as a little too rash. At the special commission appointed in 1848, to inquire into the application of wrought and cast iron to railway purposes, one engineer asserted that the working load should not surpass the $\frac{1}{10}$ of that which would fracture the material. This was an error in the opposite extreme, although an excuse may be found for its timidity, in the fact that the peculiar properties of iron were not so thoroughly understood at that time as they are now.

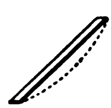
CHAPTER IV.

TRANSVERSE STRAIN.

Transverse strain may be defined as the strain brought upon a body, by a force tending to bend it transversely, or perpendicularly to its length. It is not necessary that a body should be in a horizontal position in order to undergo a strain of this nature, although, from the fact that it is usually explained with reference solely to beams and girders in a horizontal position, many naturally fall into that supposition. All long pillars similar to that represented in Fig. 6, when loaded, have a tendency to bend and assume the position shown by the dotted line, demonstrating that in addition to the crushing or compressive action of the load, a transverse strain, at right angles to the longitudinal axis of the pillar, is also exerted. The same thing happens with struts inclined at an angle to the horizon—a rafter for instance. If the rafter in Fig. 7 be uniformly loaded, it will tend to assume the position shown by the dotted line, and it is with a view to prevent this flexure, that all long rafters and struts are braced at intervals. This is the object of the introduction of the "collar" or stay in timber roofs. It will be seen hereafter, that although the collar is useful for preventing the bending of the rafters, yet it adds to the actual strain upon them. The transverse strain on the principal rafters of large roofs, can only be overcome by a skilful arrange-

Fig. 6.

Fig. 7.



ment of bracing, which has led to the adoption of the comparatively modern iron trussed roofs, so many examples of which can now be seen in our warehouses, workshops, and railway stations. While the examples in Figs. 6 and 7 are sufficient to demonstrate that a transverse strain can be produced on a pillar or beam, when placed either upright or at an angle to the horizon, yet its peculiar properties and the calculations connected with it, are always investigated upon the assumption, that the body under its influence is in a horizontal position, as represented in Figs. 8 and 9, and would have a tendency, when subjected to its action, to bend in the direction of the dotted lines. Were this course not adopted, the investigations relating to transverse strains would be complicated by those belonging to the action of the crushing

FIG. 8.

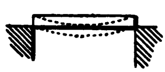
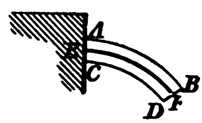


FIG. 9.

FIG. 10.



strain, and it would be very difficult to separate accurately their combined effect. Before proceeding to the investigation of the numerous problems relating to the subject of transverse strain, the bending action produced by it must be considered.

In Fig. 10, let a solid rectangular beam, A B D C, be supposed to be bent under the action of a transverse strain. Under these circumstances, the particles or fibres situated upon the upper or convex portion of the beam A B will be extended or lengthened, while those situated on the lower, or concave part, C D, will be compressed or shortened. It is manifest, that there must be a line of demarcation, or boundary, somewhere about the centre

of the beam, dividing the lengthened from the shortened fibres, and this line itself will represent the position of a layer of fibres, which are neither lengthened nor shortened, but remain, so far as their length is concerned, quite unaffected by the strain. The exact position of this line, or of the layer of unaffected particles, is in some instances a work of great labour to determine, but fortunately its approximate position in those forms of beams and girders, usually met with in practice, can be ascertained with tolerable readiness, and accurately enough for all working purposes. In the case shown in Fig. 10, let it be in the direction of the line E F. If we imagine a plane to be drawn through the line E F, so as to leave exposed a plan of the layer of invariable or unaffected fibres, then that plan represents the "neutral surface" of the beam, since it contains all the fibres unaltered in length. The line E F is therefore the longitudinal elevation of the "neutral surface," or the line representing the curve of the unaffected fibres. If a cross section of the beam be made at any point, it will cut the neutral surface, and the intersection will give us the line E F in Fig. 11, which is the neutral axis of the cross section A B C D. Very great influence upon the strength of a beam, is exercised by the position of the line E F in the cross section, and the arrangement of the material relative to it, virtually determines the proper form to be adopted for girders. A vertical longitudinal plane, M N in Fig. 12, drawn through the centre of the beam, will intersect the neutral surface, and cut all the neutral axes in the point P, which may be called the neutral point for any given section.

Those of our readers who are familiar with working drawings, will not fail to perceive that the relations

between neutral surface "line of curve" and "neutral axis" are those of plan, elevation, and section. They are

FIG. 11.

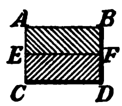


FIG. 12.

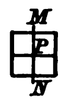
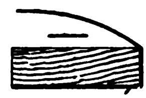


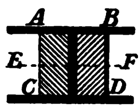
FIG. 13.



accurately represented to the same scale in Fig. 13. The load is supposed to act at right angles to the longitudinal axis of the beam, both before and after bending takes place. It cannot, mathematically speaking, fulfil both these conditions, but if the first be ensured, the second will be sufficiently close for all practical purposes. Without entering into any algebraical proof of the subject, it will be enough to state, that when the above conditions are fulfilled, the position of the neutral axis in any cross section of a beam will coincide with the position of the centre of gravity. Consequently the neutral axis of all the cross sections, or the neutral line of all square, circular, and rectangular beams, will pass exactly through their centres, and the neutral line and the longitudinal axis will coincide. With respect to the arrangement of the particles about the neutral axis, the law is, no matter what the material may be, that each particle, or each layer of fibres, exerts a resistance against a transverse strain, in direct proportion to its distance from that axis. Generally if x = any particle situated at a distance from y, the neutral axis, and x' = another particle at a distance, y', then $\frac{x}{y} = \frac{x'}{y'}$. Their respective moments will be $x \times y$ and $x' \times y'$. The conclusion to be naturally deduced from this fact is, that all the material should be placed as far as possible from

the neutral axis, a condition which when carried out gives rise to the ordinary flanged girder. If in the solid beam A, B, C, D, in Fig. 14, the material near the centre be transferred to the top and bottom, the result is a flanged girder, in which every layer of fibres on the flanges acts with nearly twice the amount of resistance in the solid beam. Since the fibres upon one half

FIG. 14.



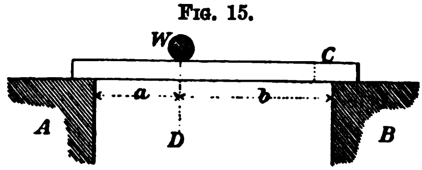
of the beam are in tension, and the others in compression, it is evident that no layer can be acting with a leverage greater than half the depth of the beam. Galileo, who was one of the first to investigate the properties of transverse strain, fell into a fatal error in imagining that the neutral axis was situate not at, or near, the centre, but at the lower edge of the beam.

At the moment of fracture, there is no doubt but that the neutral line shifts its position, and all that has been stated with regard to its coinciding with the centre of gravity, only holds so far as the safe working strain is concerned. Moreover, the neutral axis will evidently not occupy the centre of the body unless the resistances to extension and compression are identical, which is never the case. In some substances they do not differ very much, and in others they differ to a considerable extent, as in cast iron for instance. The example given in Fig. 14, where the solid beam is converted into a flanged girder, must not be regarded as indicating the best method of effecting the conversion, but only as an illustration of the manner in which the arrangements of the material, with reference to the neutral axis, affects the strengh of the body. The first flanged girders were made after this pattern, with equal top and bottom flanges, and the correct mode of distributing the material

in cast-iron girders, was not understood until the labours and exertions of the late Professor Hodgkinson discovered the true form, in which the greatest amount of strength was obtained.

The transverse strain upon a beam, resulting from a weight placed upon it, will evidently depend on the position of the weight, since the whole of the theory of horizontal beams turns upon the principle of the lever. It will be well to explain this in detail, so that a sound and accurate idea of the principle may be obtained. A general case will be taken first, and then particular examples. In Fig. 15 a beam is shown resting on two supports, A and B. A weight, W, is placed upon the beam. How is it supported, and in what proportion do the supports or abutments contribute respectively towards sustaining it? Since the beam rests upon A and B, they together support the entire weight by their upward reaction. The term reaction is virtually the same as resistance, although it sometimes perplexes the beginner, who does not exactly see the necessity of employing it. Suppose for a moment that the weight W exercises a downward pressure on the abutment A of 20 lbs., and that the abutment is only able to bear 10 lbs. It will consequently yield, and the beam will come down. In other words, its reaction or resistance is not great enough. To ensure equilibrium, that is, to prevent the beam coming down as described, the vertical reaction of the two supports must equal the total weight upon the beam, and each abutment must exert a reaction equal to the portion of the total weight

FIG. 15.



brought upon it. Any excess of resistance in the one support will not compensate for a deficiency in the other. Algebraically, if W be the total weight, P the pressure on the abutment A, and P′ that on B, R the reaction of A, and R′ that of B, then the equations of equilibrium are that $R = P$, $R' = P'$, and $(R + R') = P + P' = W$. From the position of the weight, the near abutment A evidently receives a greater portion of its downward pressure than the far one B, and, in accordance with the principle of the lever, the reaction of each is inversely as its distance from the weight, and, since action and reaction are equal and opposite, the weight W is transferred to the two supports A and B, in portions in the inverse ratio of its distance from each, or in the proportion of the two segments a and b, into which the weight divides the whole beam. For example, let $W = 20$ tons, let $a = 5$ ft., and $b = 10$ ft., then we have the proportions. Pressure on A : that on B :: 10 : 5, or the pressure on A = twice that on B. Consequently the total weight of 20 tons is transferred to the supports A and B, in portions equal respectively to 13·33 and 6·66 tons. Having disposed of the pressure on the supports, the next step is to determine the effect of the weight upon the beam itself, or the moment of the strain. The term moment may be considered as signifying the force which tends to break the beam, but must not be confounded with the term strain. The strain upon a beam, and the moment of the strain are very different, as will be explained in treating of the method of calculating the strength of girders, and determining their relative areas and proportions. So far as the weight W is concerned, two cases in connection with its effect on the beam present themselves. One is

the action at any given point C, and the other the amount of strain it develops at the point D, where it is situated. To find the moment of the strain produced by the weight W upon the beam at the point C, the reaction, or the pressure of the weight W upon either of the abutments A or B must be first determined. In working out this problem, it will be simpler to take the reaction of the support, which is nearest to the point at which the strain is required.

In other words, let the point always be situated between the weight and the abutment, of which the upward pressure or vertical reaction is employed in the calculation. Thus, were it required to find the moment of the strain upon the beam, at a point situated between W and A, the reaction at A should be used instead of that at B. In the one case it is only necessary to consider the pressure at A, but in the other it would be necessary to subtract that occasioned by W at A, from the moment found from the reaction at B, and the calculation would be more tedious.

By the moment of any force is meant the product of the force in tons, lbs., or whatever may be the unit chosen, by its perpendicular or shortest distance from the point at which the effect is required. This will be exemplified in the present instance. The vertical reaction at the abutment B is perpendicular to the longitudinal axis of the beam, or it is sufficient to assume it to be so. Its moment is obtained by multiplying it by the distance from B to C. But the reaction at B is due to the pressure exercised by the weight W, and therefore the moment of the strain C caused by W, is equal to that pressure multiplied by the distance from B to the point C. In Fig. 15 let this distance be 3′,

then the moment of the strain at C produced by the weight W is given by the calculation $\frac{20 \times 5 \times 3}{15} = 20$ tons, or since the reaction at B has been found equal to 6·66 tons, we have this moment about C = 6·66 × 3 = 19·98 tons. To solve the second question, that is, to find the moment of the strain at the point where the weight is placed, is equivalent to moving the point C to D, taking the reaction of either pier, and multiplying it by its respective distances from the weight. The reaction of the pier A is known to be equal to 13·33 tons, and multiplying this by the segment the answer is, 13·33 × 5 = 66·65, the moment of strain in tons at D, where the weight is applied. Again, the reaction at the pier B is 6·66 tons, which multiplied by the distance *b* equal also 66·6 tons. The short general rule for determining the moment of the strain at the point where the weight is applied is as follows:—"Multiply the weight in tons by the rectangle under the segments into which the weight divides the beam, and divide the product by the length of the beam, all dimensions being in feet." The calculation will stand thus: moment of strain $= \frac{20 \times 5 \times 10}{15} = 66 \cdot 6$ tons. Mathematically let W equal weight in tons, L equal span in feet of beam, and M equal moment of strain, and a, b as before: then $M = \frac{W \times a \times b}{L}$. Another question of interest here presents itself. At what part of the beam should the weight be applied, so that the moment of strain may be the greatest, or, as it is commonly called, a maximum? This will manifestly take place when the product of the rectangle under the segments is also a maximum; that is, when a is equal

to b, and the product is a^2 or b^2. When this occurs, the value of a or b becomes equal to that of half the span, equal to the square of the span divided by 4, so that by the rule already given we should have W multiplied by the square of half the span, and divided by the span. Reducing this fraction to its lowest terms, we have the moment of strain resulting from a weight at the centre obtained as follows:—"Multiply the weight in tons by the span in feet, divide by 4, and the quotient is the moment of strain in tons." Thus $\frac{W \times a \times b}{4} =$ strain, but $a = b = \frac{L}{2}$. Therefore $\frac{W \times L^2}{4\,L} = \frac{W\,L}{4} =$ moment of strain at centre. In the example, suppose 20 tons to be placed at the centre, then the calculation $\frac{20 \times 15}{4} = 75$ tons. The same result can be deduced by the principle of the lever which governs all the strains upon horizontal beams. The weight W at the centre is transmitted in equal portion to the supports A and B, which react upon the beam, and a strain of $\frac{W}{2}$ is impressed upon it in consequence, and the leverage with which it acts is half the length of the beam, since the weight is in the middle. If we multiply half the weight by half the span, the moment of strain becomes equal to the weight multiplied by the span, and divided by 4, as before.

CHAPTER V.

WEIGHTS UNUNIFORMLY DISTRIBUTED.

In the last chapter, the action of a single weight, in all the various effects it was capable of producing upon a beam, was considered, and the reactions of the supports, the moment of the strain at any point along the beam, at the place where the weight was supported, and the point at which the weight should be placed to produce the greatest possible moment of strain, were all determined. The two most important results to be remembered are, that the greatest strain is produced on a beam by a weight when it is situated at the centre, and when a weight is placed at any point, the strain it produces at that point is greater than at any other. This can be readily shown. In Fig. 16, let the weight W divide the beam into the segments a and b. The strain at any point C is equal to the reaction of the abutment A multiplied by the leverage or distance between it and the point C. As the leverage increases, so will the strain, but it attains its greatest value, when it equals the distance between the weight and the support A, that is, when it equals the segment a. Consequently the greatest strain produced by the weight W is at the point where it is applied. Having investigated all the cases relating to the strains induced by a single weight upon a beam, the effect of two will be considered, and then the

Fig. 16.

general problem relating to a beam loaded with any number of weights.

In Fig. 17 let the beam be loaded with two weights W and W′, the first dividing it into segments a and b, and the second into those of c and d. Let it be required to determine the total moment of strain exercised by the weights at the point C. There are two methods of proceeding in this instance. We may either take the separate moment of strain produced by each weight at the point C, and then add them together for the total moment, or we may take the resultant of the two weights, and proceed upon the supposition that only one weight equal to the resultant is placed upon the beam. Whatever method may be selected, the final result will be the same. To commence with the first. The weight W produces a strain upon the point C equal to its reaction at the support B multiplied by the distance from B to C. Similarly for the weight W′. As an example, let $W = 20$ tons, $W' = 10$ tons, $a = 5$, $b = 10$, $c = 5$, $d = 10$. The reaction of W at B is 6·6 tons, and the moment of strain at C $6{\cdot}6 \times 3 = 19{\cdot}8$ tons. The reaction of W′ at B is also 6·6 tons, and consequently it gives another strain at $C = 19{\cdot}8$ tons, therefore the whole moment of strain at the point C equals 39·6 tons. It is easy to perceive that since the value given to W′ is exactly half that of W, while its distance from B is exactly double that of W; therefore the reaction at B and the moment of strain at C will be equal. The other method consists in first finding the position of the resultant of the two weights W and W′ on the beam, and then proceeding as before. This is a common problem

FIG. 17.

in determining centres of gravity which are all based upon the problem of the "resolution of forces." The two weights W and W′ may be regarded as two forces acting upon the beam, and it is required to find where their resultant should be placed, in order to equal their combined effect. Suppose its position to be represented by that of R in Fig. 18, where the length of the line equals the distance between the weights in Fig. 17. As the weights are unequal, it is clear that the resultant will divide the line into two unequal segments. Make these equal to x and y respectively. The position of R is found by the following proportion: $x : y :: W' : W$; but as the distance between the weights on the whole length of the line is 5′, the proportion becomes, $x : (5-x) ::$ $10 : 20$, from which we obtain $20\,x = 10\,(5-x)$, or $2x = 5 - x$. Solving for x we have $x = \frac{5}{3} = 1{\cdot}66$. That is, the position of the resultant or sum of the two weights is 1·66′ from the weight W, and its actual position on

FIG. 18.

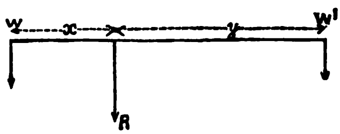
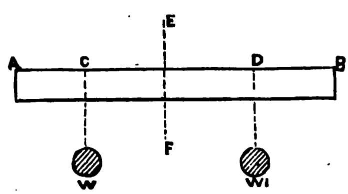


FIG. 19.



beam is therefore 6·66′ from the abutment A, and consequently 8·34′ from B. The position being known, it is easy to determine the strain at C. The value of the resultant is 30 tons, and its reaction at B, multiplied by the distance from B to C, will give the moment of strain. The calculation will therefore stand $\frac{30 \times 6{\cdot}6 \times 3}{15} = 39{\cdot}6$ tons, the same result found by the

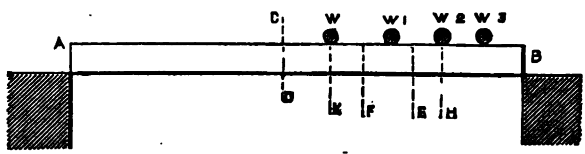
other method. Our readers will, of course, choose for themselves which method they prefer, but to thoroughly understand the subject they should make themselves acquainted with both. It is not often that a beam is loaded in this manner with two weights, but one remarkable instance occurred at the raising of the Britannia tubes, which is sufficient to prove to young engineers, that they can never know when an unusual problem may be presented to them to solve. If they only devote their study to the ordinary calculations, and the getting up by rote of numerous hackneyed and frequently inapplicable formulæ, they will find themselves at a loss, when a question arises which demands a sound and accurate knowledge of the principles and rules, upon which the strength of beams, under every possible circumstances and conditions of loading, is required. The example of a beam being subjected to a pair of weights, as just described, occurred in the crosshead of the presses employed to raise the magnificent tubes now spanning the Straits of Menai. The case, as put in "The Britannia and Conway Tubular Bridges," is shown in Fig. 19, where A B represents a beam, loaded similarly to the crosshead, by two weights W and W′, the moment of the strain at the centre being required. Since A C = C E = E D = D B, the span of the beam may be made equal to 4, each of these subdivisions being equal to unity. The weights W and W′ equal each 450 tons, and the moment of the strain they exercise at any point, such as, in this instance, at the centre of the beam, is equal to their reaction at the abutment situated beyond the point, multiplied by the distance between that abutment and the point. Thus the portion of W transferred to the support B, or its reaction, there is evidently one-quarter

of its amount, which is equal to $112 \cdot 5$. This, multiplied by the distance E B, equal to 2, gives the moment of strain equal to 225 tons. The same effect is produced by the action of the weight W′, and therefore the total strain at E F, the centre of the crosshead, is 450 tons. It may, perhaps, be well to point out that the answer can be obtained equally accurately, by taking the reaction at the other support for each weight. Thus the reaction of W at A is $337 \cdot 5$ tons, which, multiplied by the distance A E, equal to 2, would give a moment of strain at E equal to 675 tons, but from this must be deducted the weight at W, multiplied by the distance C E, which is 450×1, and $675 - 450 = 225$ tons. By taking the reaction of W′ at the support B and proceeding in the same manner, we shall obtain another strain of 225 tons, making the total, as before, 450 tons. Again, as E F is the position of the resultant of the two weights W and W′ equal together to 900 tons, it might be expected that in accordance with the example given above, the strain upon the centre found by those means would equal that already determined. The calculation by the rule for a central strain would be $\frac{900 \times 4}{4} = 900$ tons, or exactly double that found by taking the action of each weight singly. There are evidently here some points to be attended to. In the first place, the method of finding the strain at any point C in Fig. 17, by using the resultant of the two weights W and W′, will not hold when the point is situated between the weights, as in Fig. 19, whether that point be at the centre or elsewhere. Unless the two weights are both situated upon the same side of the point of strain, they cannot be accurately represented by a single resultant, since it is

necessary that the reaction of both abutments or supports enter into the calculation, which cannot be done with a single weight represented by the resultant at E F in Fig. 19.

To place this matter clearly before our readers, let it be required to find the strain upon the centre of the beam A B in Fig. 20, where four equal weights are placed

FIG. 20.



at equal distances apart upon the half C B of the beam A B. Let each weight equal 10 tons, and let the whole beam be 50′ in length. In this case the resultant of 40 tons will act at the centre of gravity E of the half-beam at a distance of 12·5 ft. from the abutment B. To find the moment of strain at the centre we take the reaction at A and multiply it by A C, thus: $10 \times 25 = 250$ tons; or working from B $(30 \times 25) - (40 \times 12{\cdot}5) = 250$ tons. In order to give another instance besides that in Fig. 19, where this method is fallacious, let the strain be required at the point F, 16′ from B, from the action of the same four weights. Proceeding similarly from the support A, the calculation stands $(10 \times 34) = 340$ tons. Now we will ascertain the correct strain, which will be composed of the separate action of the weight W, situated upon one side of the point F, and that of the three others, W^1, W^2, W^3, situated upon the other. The action of the weights W^1, W^2, W^3, may be reduced to a single resultant acting at their centre of gravity, since they are placed upon the same side of the point F. This position

is at H, at a distance of $7 \cdot 5'$ from the abutment B; consequently the calculation for the moment of the strain produced at F by the three weights, will be the reaction of 30 tons at A, multiplied by A F, equal to $(4 \cdot 5 \times 34) = 153$ tons, or from $B = (25 \cdot 5 \times 16) - (30 \times 8 \cdot 5) = 153$ tons. The moment of strain also due from the one weight W, situated 20′ from the abutment B, will be expressed by $(4 \times 34 - 10 \times 4) = 96$ tons, and the whole moment of strain at the point F, due to the action of the four weights, will be equal to 249 tons, instead of 340 tons, as found by the erroneous method. To prevent the possibility of mistake, it would therefore be prudent to adhere to the general method of taking the action of each weight singly, and then adding them together for the total moment of strain. If a number of weights be upon each side of a point, at which the strain is to be determined, the position of the resultant of all the weights that lie upon separate sides may be ascertained, and the problem treated as if there were only two weights upon the beam, one upon each side of the point. This is virtually the manner in which the last question has been treated. Mathematically, to find moments of strain at centre of beam C D and at F, let $R =$ the resultant of the four weights acting at the point E.

Let $L =$ the span of beam, and M and M′ the respective moments $M; = \frac{R \times EB}{2}$. For the moment at F let $R' =$ the resultant of the three weights acting at point H, then

$$M' = \left\{ \frac{R' \times HB \times AF}{L} + \frac{W \times AK \times BF}{L} \right\}.$$

Instead of finding the resultant of all the four weights, W, W^1, W^2, W^3, in Fig. 20, their separate action might

be taken, and then the sum would give the total moment of strain at the centre. It is easy to perceive that the calculation would be simply the summation of a series, multiplied by the common quantity of half the length of the beam, and consisting of the sums of the reactions at the support A into the length A C. The strain would be expressed thus: $25(4+3+2+1) = 10 \times 25 = 250$ tons as before.

The extension of the case where a beam is completely loaded with any given number of weights, constitutes what is termed a uniformly distributed load, where a beam is supposed to be loaded with a certain weight per foot run. There are, nevertheless, some peculiar features attending the uniform distribution of a load which do not belong to single weights. Referring to Fig. 19, it will be observed that the moment of the strain, produced at the centre of the beam, by two weights together equal to 900 tons, and situated at equal distances between the centre, and the ends of the beam, was exactly half that produced when the whole 900 tons were placed at the centre. This deserves especial consideration, for upon it is based the relation that exists between a number of weights uniformly distributed over a beam, and the same number of weights placed at the centre. From an inspection of the figures, it is clear that from their position, the two weights of 450 tons may be each regarded, as the resultant of a number of weights, uniformly distributed over each half of the beam, and acting at their centre of gravity. Consequently, to all intents and purposes the beam in Fig. 19 may be regarded as uniformly loaded with a distributed weight of 900 tons, and producing under these conditions a moment of strain at the centre equal to 450 tons. From this example can be deduced

the ratio of the moments of strain produced at the centre of a beam, by a weight distributed uniformly over it, and that weight collected at the centre. This proportion may be expressed thus,—the moment of strain, or the actual strain, at the centre of a beam, produced by any number of weights uniformly distributed over it, is exactly one-half that which would be produced, if the total amount of the separate weights were collected at the centre. For example, a loan of 100 tons situated at the centre of a beam 20′ in length would, by the rules already given, produce a moment of strain equal to

$$\frac{100 \times 20}{4} = 500 \text{ tons.}$$

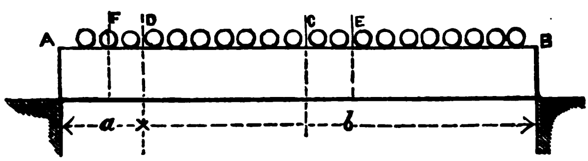
But if this weight were spread uniformly over the beam, that is, at the rate of 5 tons per foot run, the moment of strain at the centre would only be 250 tons.

CHAPTER VI.

WEIGHTS UNIFORMLY DISTRIBUTED.

THE action of a distributed load will be treated in the same manner as in the preceding instance, by first considering the general case, and then directing attention to those particular ones most usually met with in practice. In Fig. 21, let A B be a beam loaded uni-

FIG. 21.



formly with a weight W per foot run. What will be the strain at the point D? From what has been already stated, the resultant of the weights upon one side of the point, may be regarded as acting as a single weight, at the centre of gravity of the distance from B to D, and the resultant of those upon the other side, may be considered in a similar light. Considering first the weights that lie to the right of D, let their resultant act at the point E, then the moment of strain at D is equal to the reaction at A, multiplied by the distance A D, or the length of the segment a. Similarly the moment of strain from the weights upon the left of D, is equal to the reaction at B of their resultant acting at the point F, multiplied by the distance B D, or the segment b. The

total moment of strain at D will therefore be the sum of these two separate moments. As an example, let the beam be 200′ in length, and loaded with a uniform load of 1 ton per foot run. It is required to find the moment of strain at the point D. From the problem, the sum of the weights upon the right of D equals 170 tons, and their resultant acts at the point E at a distance of 85′ from the support B. Consequently the reaction at A is expressed by $\frac{170 \times 85}{200} = 72 \cdot 25$ tons. Multiplying this by the length of the segment $a = 30'$, the strain at D = 2167·5 tons. From the other weights, which equal altogether 30 tons, the resultant of which acts at the point F, at a distance of 15′ from A, the reaction at B equals $\frac{30 \times 15}{200} = 2 \cdot 25$ tons. This multiplied by the distance B D, or length of segment, $b = 170'$, gives a moment of strain at D equal to 382·5 tons. The total moment of strain therefore produced at D by a uniform load of 1 ton per foot run upon the beam is (2167·5 + 382·5) = 2550 tons. The same result might have been obtained by taking the individual reaction of each separate weight, and adding them together; but it would manifestly be a very tedious operation in the example in Fig. 21, nor is there any necessity for so doing. There is another method of determining the moment of strain at D, which is shorter and simpler than that described, and moreover enables a general rule to be arrived at.

Since the load is uniformly distributed at the rate of 1 ton per foot run, the total load may be put equal to 1 ton, multiplied by the length of the beam, and the reaction at each abutment is equal to half the total

load. But the strain at the point D is equal to the reaction of the half load at A, minus the moment of the weights situated between A and D, which acts at their centre of gravity, F. The calculation will therefore stand $(100 \times 30 - 30 \times 15) = 2550$ tons as before. The rule to find the moment of strain at any point produced by a distributed load is, "Multiply the total weight by the segments into which the point divides the beam, and divide the product by twice the length." To prove this the calculation will be as follows:

$$\frac{200 \times 30 \times 170}{2 \times 200} = \frac{5100}{2} = 2550 \text{ tons},$$

the same result already obtained. Mathematically, the identity of the two methods and the deduction of the rule may be thus proved. Let W = load per foot run, W^1 = total load, L = span, and a and b as in the figure. The moment of strain due to weights upon right of $D = \frac{W \times b \times BE \times a}{L}$, and of those on the left, $\frac{W \times a \times AF \times b}{L}$. Therefore the total amount of strain at $D = \frac{(W \times ab \times BE + W \times ab \times AF}{L}$.

But by the problem $BE = \frac{b}{2}$ and $AF = \frac{a}{2}$, and the moment of strain becomes equal to $\frac{W a b^2 + W a^2 b}{2 L}$ $= \frac{W a b (a + b)}{2 L}$. But $W (a + b) = W^1$ = total load, and the formula becomes, putting M for moment of strain, $M = \frac{W^1 \times a \times b}{2 L}$, which is identical with the rule

given above. The identity of the two methods may be thus proved:

$$M = \frac{W \times L \times a}{2} - \frac{W \times a \times a}{2} = \frac{W \times a}{2}(L - a).$$

But $(L - a) = b$ and $W = \frac{W^1}{L}$, therefore substituting these values in the above equation, $M = \frac{W^1 \times a \times b}{2 \times L}$, the result obtained by the other method. From this it follows that the strain produced at any point of a beam, by a uniformly distributed load, is one-half that produced by the same load collected at the centre, and that the strain is proportional to the rectangle under the segments. In the same example, let it be required to find the moment of strain at the centre, from the same load distributed uniformly. In this case the segments are each equal to half the line, and the moment will be equal to $\frac{200 \times 100 \times 100}{2 \times 200} = 5000$ tons.

The general formula already given is $M = \frac{W^1 \times a \times b}{2\,L}$, but $a = b = \frac{L}{2}$, and $M = \frac{W^1 \times L^2}{8 \times L} = \frac{W^1 \times L}{8}$.

This formula might also be readily deduced from the second method, for

$$M = \frac{W^1}{2} \times \frac{L}{2} - \frac{W^1}{2} \times \frac{L}{4} = \frac{W^1\,L}{4} - \frac{W^1\,L}{8} = \frac{W^1 \times L}{8}.$$

For the moment of the strain at the centre the rule will be: "Multiply the total uniformly distributed load by the span of the beam, and divide the product by 8." Thus $\frac{200 \times 200}{8} = 5000$ tons. By the rule previously given for finding the moment of strain at the centre, pro-

duced by a weight placed at the centre, the moment is exactly double what it is in the present instance. It is therefore assumed in practice, that any girder or beam will bear twice the load uniformly distributed over it, that it will at the centre. This assumption is not correct in all cases, as will be pointed out hereafter. The strains at any two points of a beam, being to one another in the proportion of the rectangle under the segments, into which they divide the beam, the strain at the centre of Fig. 21 should be to that at the point D in the ratio of a multiplied by b to the square of half the length of the beam. In other words, the following proportion should exist: 2550 : 5000 : : 5100 : 10,000, that is $(255 \times 1000) = (500 \times 510) = 255{,}000$.

Any problems that may present themselves, respecting the moment of strain upon horizontal beams supported at both ends, can now be readily solved. Sometimes instances will arise where a beam may be not only uniformly loaded, but have a weight besides placed at some point of it. In all cases of this description, the principle to be observed is to find the strain, due to each weight regarded separately, and then take the sums to obtain the total effect. It would be impossible to investigate all the numerous cases and combinations that might arise; but those who make themselves thoroughly acquainted with what has been already laid before them, will have no difficulty, with the exercise of their own brains, in solving any questions that may present themselves. If it were possible to determine at once all the problems that might come before the engineer, he would have little or nothing to do, and but very little credit would be due to him, for merely calculating arithmetically, what everyone else who had read the same treatise could

work out equally well. It is in the exercise of his own ingenuity, scientific attainments, and practical experience that he ultimately hopes to distinguish himself, and not by slavishly copying existing precedents, and never going beyond his predecessors' limits. The subject of the strains upon cantilevers, or beams fixed at one end and unsupported at the other, is equally important with that already considered. Let the general case be represented in Fig. 22, where the weight W is situated at any point upon the beam, and the moment of strain is required at the point C. This is simply an example of leverage, and the moment of strain at C equals the weight multiplied by its distance from the point. If the weight be shifted to the extremity B, the moment of

Fig. 22.

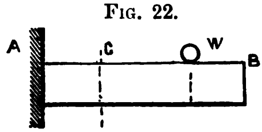
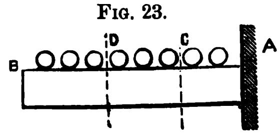


Fig. 23.



strain at C will be a maximum, since the distance B C is also one. The greatest moment of strain that can come upon the beam will be at A, when the weight is at B, and will therefore be equal to the weight multiplied by the length of the beam. For a load uniformly distributed over the beam, the moment of strain is found equally readily. In Fig. 23 let A B be a cantilever loaded uniformly per foot run, and let it be required to find moment of strain at C. As in the case of beams supported at both ends, all the weights between the point C and the end B of the beam may be taken as concentrated in a resultant acting at D, their common centre of gravity. The moment of strain will be therefore equal to the sum of the weights between B and C

multiplied by the distance D C. Since the weights are all transferred to one support, those situated between the point C and the support A have no influence upon the moment of strain at C. Let the beam be 8′ long, and each weight equal to 1 ton; moment of strain at C $= 6 \times 3 = 18$ tons. If these six weights be shifted to the end B of the beam, the strain at C will manifestly be equal to $6 \times 6 = 36$ tons, or just twice the amount in the former case. It is apparent that a similarity exists between the ratio of single weights, and distributed loads, in half-girders fixed at one end, as well as in those supported at both. The greatest strain will take place in the beam A B at the point A, and will be equal to the total load uniformly distributed multiplied by half the length of the beam, and is consequently exactly half what it would be were the weights all collected at the free end of the beam. In our example, moment of strain at A $= 8 \times 4 = 32$ tons. Were the 8 tons placed at B, the moment of strain would become $8 \times 8 = 64$ tons. The same analogy exists between the two descriptions of beam, for we thus see that a cantilever will bear twice the load uniformly distributed over it, that it would collected at its extremity. Let W equal weight per foot run, L the distance of the point C from the free end of the girder, then putting M for the moment, $M = \frac{W \times L^2}{2}$, but when $L = L^1 =$ total length of beam, $W \times L^1 = W^1 =$ total load, and $M = \frac{W^1 \times L}{2}$ for a distributed load. When W^1 acts at end of girder $M = W^1 \times L =$ maximum strain for a load at the end of beam.

As the moment of strain is to be regarded as the at-

tacking force, or force tending to break the girder at any point, it is evident that to arrive at the actual strength of the structure, its resistance must be determined. The two forces can then be equated, and a rule deduced for the actual strength of the girder, and the proportions it ought to possess. The rules given demonstrate that the strength of a girder, or its capabilities of resistance to resist fracture, are inversely as its span between bearings. The load and the quantity of metal in an iron beam being the same, if its span be doubled, its strength will be halved. If a girder 20′ long will just break with a weight of 100 tons, it will also yield with a weight of 50 tons, if its length be increased to 40′, and the other dimensions remain constant. The strength of any beam will also be in direct proportion to the quantity of material in it. If the span, weight, and depth remain constant, a beam having 24 square inches of sectional area, will bear twice the weight that one would which had only 12 square inches. The strength of a beam is therefore directly proportional to its sectional area, and inversely proportional to its span or distance between supports. But there is another very important element concerned in the strength of girders, *viz.* the depth. Supposing the load and sectional area constant, the strength of the girder will be directly as the depth, and inversely as the span. The manner in which the sectional area and span effect a beam, is too obvious to require any proof, but the effect of the depth is not so readily perceptible. The form of girder in almost universal use at present among engineers is the flanged type, of which a cast-iron specimen is the simplest, and represented in Fig. 24. It consists of two distinct parts—the flanges and the web; the top flange being

denoted by A, the bottom by B, and the web by C. When the girder is loaded, strains are developed in both the flanges and web, but at present we shall confine our attention to those induced in the flanges, and suppose the girder, as it is invariably assumed to be in practice, loaded with a uniformly distributed weight which includes its own. Let the girder be shown in Fig. 25, loaded

FIG. 24.

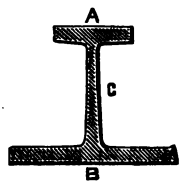
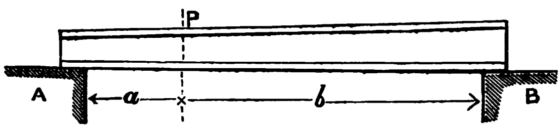


FIG. 25.



uniformly per foot run, and let it be required to find the strain upon the flanges at the point P, dividing the girder into segments *a b*. This strain will be horizontal in direction, will be equal upon both the upper and bottom flange, but will not be of the same character for each. It will be compressive in the top flange, and tensile in the lower—that is to say, it will tend to shorten the fibres in the upper, and lengthen them in the lower flange. This has been already explained, when treating of the neutral axis of bodies in a preceding chapter. As the horizontal strains are equal on both flanges, but the resistance of cast iron to compression and tension is widely different, it is for this reason that the lower flanges of cast-iron girders are considerably larger than the upper. To find the strain at the point P, the forces that act upon one side of it, and those acting upon the other must be considered. The forces acting upon one side of the point P, are the reaction at A, multiplied by

the length of the segment a. The force acting in opposition to this moment of strain, which tends to break the girder at the point P, are the actual strain, the depth, and the weight of the segment a, which acts at its centre of gravity, $\frac{a}{2}$. This will be more easily comprehended by taking an example, and bearing in mind that the solution is general for all beams and girders loaded in the same manner. Let the span of the girder, or $(a+b)$, equal to $50'$; let the load, including the weight of the girder, be at the uniform rate of 1 ton per foot run, making the total load equal to 50 tons. Put the depth equal to $5'$, and let $a = 10', b = 40'$. The moment of the force tending to break the girder at P will be $25 \times 10 = 250$ tons. The forces in opposition will equal the actual strain multiplied by the depth, plus the weight of the segment, into the distance of the centre of gravity of a, from the point P, and the strain will therefore equal

$$\frac{250 - 50}{5} = 40 \text{ tons.}$$

Let W equal total load L = span of girder = $(a + b)$; D = depth; S = horizontal strain on flanges; and P = weight of segment a.

Then by the proposition we have

$$\frac{W}{2} \times a = S \times D + \frac{P \times a}{2},$$

from which

$$S = \frac{W \times a}{2 \times D} - \frac{P \times a}{2 \times D} = \frac{a}{2}\left(\frac{W - P}{D}\right).$$

But $P = \frac{W \times a}{(a + b)}$, consequently,

$$S = \frac{a}{2 \times D}\left(\frac{W(a+b) - Wa}{a+b}\right) = \frac{W \times ab}{2DL}.$$

The limits of the value of a and b are $a = o$, $b = L$,

$a = b = \frac{L}{2}$. In the former instance $S = o$, and in the latter $S = \frac{W\,L}{8 \times D}$ = strain at the centre of a girder under a uniformly distributed load. Each of the flanges A and B will therefore be subjected to a horizontal strain of 40 tons at the point P, that upon the upper flange being of compression, and upon the lower of tension. The general rule for finding the actual strain upon any point of a girder, due to a uniformly distributed load, may be thus stated: "Multiply the total load in tons by the rectangle under the segments in feet, into which the point divides the girder, and divide the product by the span in feet multiplied by twice the depth, also in feet." The calculation for the strain at P will therefore, in accordance with this rule, be $\frac{50 \times 10 \times 40}{2 \times 5 \times 50} = 40$ tons, as before. For the strain at the centre, the segments then become each equal to one another—equal to half the span, and the rule is: "Multiply the total weight in tons by the span in feet, and divide the product by eight times the depth likewise in feet." It may be mentioned, once for all, that multipliers and divisors must always belong to similar units, that is, if feet be used as a multiplier they must also be used as a divisor.

The student should always make his calculations in clear and bold figures, and step by step, so that, should the answer not come out at the first trial, he would be able at a glance to discover whether the mistake is in the reasoning or theory of the problem, or simply an arithmetical one. Probably the best method of making calculations, or, at any rate, of preserving the results, is to make them in a book, when they can always be re-

ferred to. As it is a very common occurrence, in the calculation for bridges, roofs, and other engineering structures, for the same, or nearly the same, example to arise over again, a reliable calculation that has been worked out in all its details, and practically executed, is of much value, and saves subsequently both time and labour.

CHAPTER VII.

GRAPHIC METHOD OF DETERMINING STRAINS.

The strains upon beams have hitherto been determined by the method of calculation, which, if correctly performed, must give a result mathematically true. There is, however, another method which may be employed in many instances with less trouble and less mental exertion, known as the geometrical or graphic method. In certain cases, where trussing and bracing of a complicated and intricate nature are introduced, this is the only method practically applicable for determining the strains, and it will be had recourse to, when treating of those examples of iron construction. The whole value of this elegant and simple method of arriving at the determination of strains, depends absolutely upon the accuracy with which the diagrams are drawn to scale. It is of course premised that the reasoning is sound, and that it is only necessary to work out the details of the problem. A common objection to this plan is, that it is not susceptible of that extreme precision which is afforded by analytical calculation. This objection is far more specious than real, and is very often an excuse put forward by those who are careless and slovenly with the pencil and scale, and besides, are not sufficiently acquainted with the theory of the method to work it out successfully. There is another great advantage connected with the graphical method of calculating strains, which is, that something more than a superficial knowledge of their nature and

action is necessary. In working out calculations by its aid, the strains cannot be obtained all at once, but the operation must be commenced *ab origine*, and the various lines in the diagram obtained in succession. This is requisite, since the finding of the direction and value of any particular line, depends upon the accurate determination of those preceding it. It is not so with mere calculation, since anyone might take one of the rules already given and arrive at the strain upon a girder by its means, without having the least idea of the data upon which it was founded, or in what manner the position of the load affected the resistance of the beam.

The calculation of the horizontal strains upon the flanges of girders, when those upon the web are neglected, is so simple, that the graphic method is rarely employed, but in order to show its application, and as an introduction to the principle, we have selected an example in Fig. 25. It is drawn accurately to scale, so as to demonstrate that the method is susceptible of every degree of precision necessary for practical purposes, and if a diagram, so small as the one in the cut, will give correct results by the scale, there is no need of pointing out that a working diagram will do so *a fortiori*. In Fig. 25, A, B, E, F, is the skeleton outline of a flanged girder, similar to that represented in Fig. 24 in the last chapter. The span is 50′, the depth, 5′, and it is supposed to be loaded with a weight of 25 tons at the centre. It is required to determine, by the graphic method, the horizontal strains upon the flanges at every 5′ along the span, and to check the results afterwards by the rules already given for such cases. The first step is to select a scale upon which to plot and measure the strains. This must depend in a great measure upon the size upon which

the diagram is made, bearing in mind that the larger the scale the more accurate will be the results. At the same time there is no use in going into the extreme and selecting a scale extravagantly large, to the creation of a great deal of inconvenience and chance of error in the subsequent prolongation of the lines. A very large scale is not so necessary, as the accurate plotting of the strains upon that which may be chosen. The magnitude of the scale will never compensate for inaccurate plotting, and, as a rule, a medium proportion with careful attention to the ruling of the lines will be found preferable to a scale of excessive dimensions. In Fig. 25 the scale for plotting and measuring the strains is 20 tons to the inch. The weight of 25 tons at the centre will cause a reaction of 12·5 tons at each abutment. To the given scale plot the lines A C, F D, each equal to 12·5 tons, and join the points C and D by the horizontal dotted line C D. It will be sufficient to determine the strain for one-half of the girder, since those upon the other half will be the same. From the point A, through each 5′ distances marked 1, 2, 3, 4, 5, upon the upper flange of the girder, draw the lines A G, A H, A J, A K, A L, cutting the horizontal line C D in the points G, H, J, K, and L. The horizontal strain upon the flange at the points 1, 2, 3, 4, 5, will be measured by the lines C G, C H, C J, C K, C L, and will be respectively equal to 12·5, 25, 37·5, 50, and 62·5 tons. The line A K drawn through the point 4 is a proof of the accuracy of the diagram, as it exactly cuts the centre of the line C D, the half of which upon the same scale as the strains would be equal to 50, the number of tons representing the strain at the point 4. These results may be checked by the method by calculation. Since the weight is at the centre

the strain at the points 1, 2, 3, 4, 5, will be equal to the reaction at A multiplied by the distances B^1, B^2, B^3, &c., and divided by the depth. Since the reaction is constant and the depth also, we may make $\frac{12 \cdot 5}{5} = 2 \cdot 5$, a common multiplier. The strains will therefore vary as the distances multiplied by this common quantity. Their respective values will be therefore equal to $5 \times 2 \cdot 5$, $10 \times 2 \cdot 5$, $15 \times 2 \cdot 5$, $20 \times 2 \cdot 5$, and $25 \times 2 \cdot 5$, the separate products of which will give the results already obtained, by the scaling of the lines in the diagram. Instead of plotting the reaction upwards at A, the strains may be determined by dividing the lower flange into the necessary number of parts, plotting the 12·5 tons downward from E equal to the line E K′, drawing the horizontal line K′ N, and taking P as the origin, draw in the lines P G′, P H′, P J′, P K′, P L′. The horizontal strains upon the points 1, 2, 3, 4, 5, will be found by scaling the respective lines P′ G′, P′ H′, P′ J′, P′ K′, and P′ L′, which are equal to those, giving strains of similar amount in the top flange. The strains in the top and bottom flanges are therefore equal in amount, although different in nature, those upon the upper flange being strains of compression, and those upon the lower, strains of tension. The diagram also demonstrates the proof of a previous statement, that the strain occasioned at the centre of a girder, by a given load situated at the centre, is equal to the strain occasioned at the same point by double the given load uniformly distributed. If the line C L or P′ L′ be measured it will be found equal to a strain of 62·5 tons, due to the presence of a weight of 25 tons placed at the centre. Assuming that the girder is uniformly loaded with twice this weight or 50 tons, we have by

the rule already given, the strain at the centre equal to $\frac{50 \times 50}{8 \times 5} = 62 \cdot 5$ tons the formula being $S = \frac{WL}{8\,D}$.

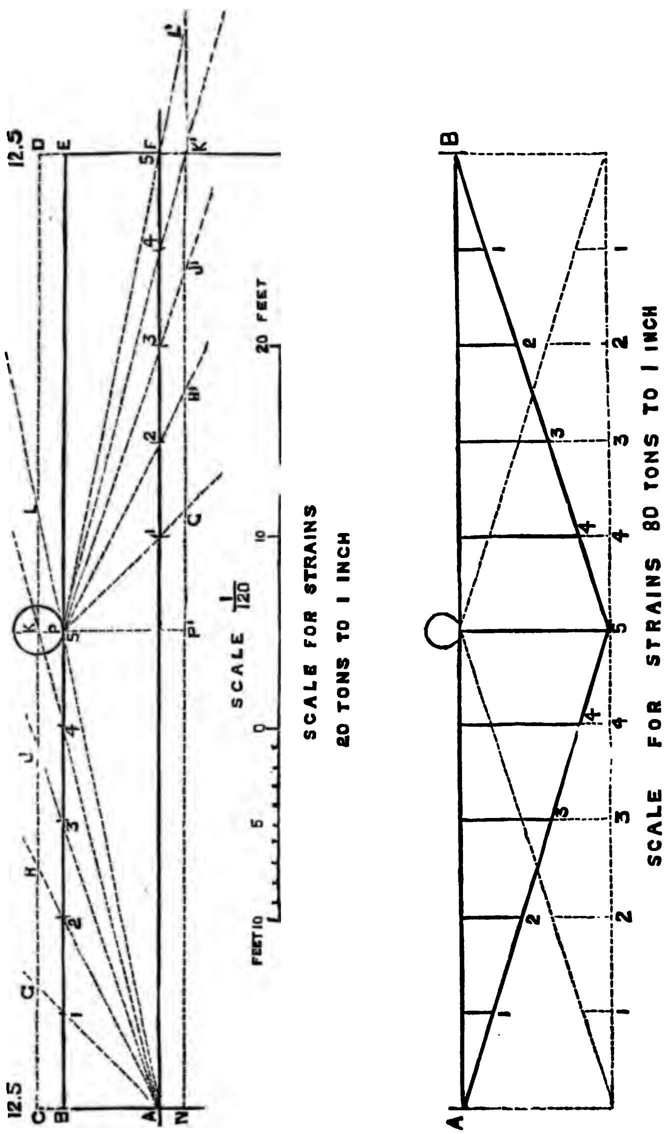


Fig. 26.

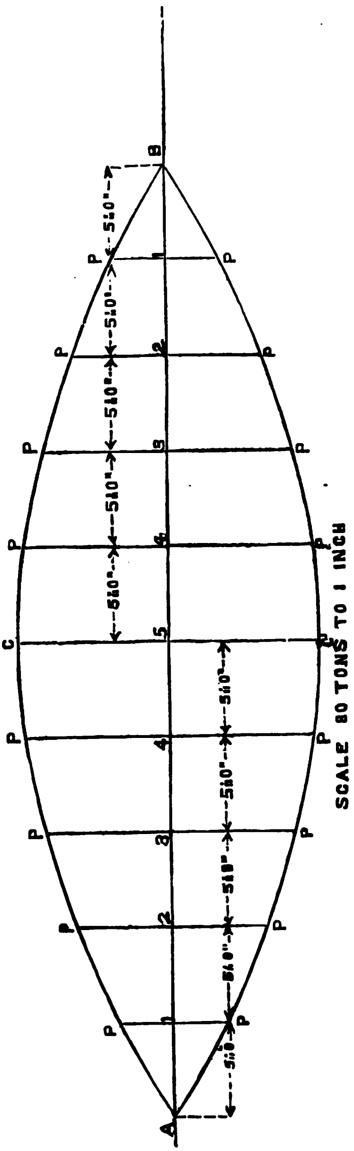
The diagram in Fig. 25 will indicate how the best form of girder in elevation subjected to a load at the centre may be arrived at. It is, that the strength of a girder is directly as the sectional area and the depth, and inversely as the span. But it is apparent from Fig. 25, that the horizontal strains upon the flanges increase from the ends, where they are equal to zero, towards the centre where they reach their maximum amount, and their increase is proportional to their distance from the end, that is in the proportion of the lines C G, C H, &c. Assuming the sectional area of the flanges, or their breadth into their thickness, to be constant, it is necessary, in order to resist in the most economical manner, the strains which vary throughout the length of the girder, that the remaining dimension, upon which its strength depends, must vary also. That dimension is its depth. To discover the ratio in which the depth varies we must suppose one of the flanges, the top for instance, to be horizontal, as represented in Fig. 26, and divided into spaces 5′ apart. With any given scale, plot the strains already determined at the points 1, 2, 3, 4, and 5. If this be accurately done, it will be found that the line joining the point A with 5, will pass through all points indicating the amount of strain. Performing the same operation for the other half, the elevation of the girder, under the conditions assumed, will be that of a triangle. The same result could have been readily arrived at by analytical reasoning, but it might not have been so clear to some of our readers. If the lower flange be taken as the horizontal one, the same process can be carried out, and the triangle will be represented by the dotted lines in Fig. 26. Were the flanges of sufficient strength to withstand the transverse strain, the inter-

mediate part of the girder, or the web, might be removed, and it would constitute the simplest example of a truss. If it be assumed that the depth is constant, the sectional area and consequently the breadth of the flange must vary, since in cast-iron girders the thickness ought to be uniform, or very nearly so, throughout the whole length. For a girder, therefore, in which the depth is constant, the shape of the flange on plan, will be that of two triangles, with the bases meeting at the centre, supposing the weight to be placed there. This may be easily ascertained by drawing a line, representing in plan, the centre line of the girder, and plotting off the strains previously determined. Upon joining all the points upon each side of the half span, which, if the diagram be correct, will all lie in the same straight line, the plan of the girder will be accurately defined.

The next and more usual case presenting itself is that of a distributed load. Where cast-iron girders are used the depth is frequently maintained constant, with the exception of a little rounding-off at the extremities of the upper flange. The thickness of the flanges is also constant, since castings having unequal thickness in their component parts, are to be sedulously avoided. It would, moreover, in the case of cast-iron girders, be impossible to alter the thickness of the same flange. Supposing the depth constant, the strains will necessitate a varying area at different points of the flanges, similarly to the instance just considered. The area being equal to the breadth into the thickness, and the latter quantity being a constant, the breadth of the flanges is therefore the varying dimension. The horizontal strain upon any point of a girder, uniformly loaded with a distributed weight, will vary as the rectangle under the segments into which

the point divides the span of the girder, and consequently the breadth of girder must vary in this proportion. If the girder, therefore, be divided into segments *a* and *b* along its whole length, the breadth at any one point, should be to that at any other point, as the product of the four different segments, taken two by two. In Fig. 27 is represented a plan of a girder designed to carry a uniformly distributed load, the span being 50′ and the load 1 ton per running foot. At every 5′, at the points 1, 2, 3, 4, and 5, the strains may be calculated from the rules given, and will be found to be 22·5, 40, 52·5, 60, and 62·5 tons respectively. Upon any given scale, plot off these strains, or any subdivisions of them, upon lines, drawn perpendicular to the axis A B of the girder, through the points 1, 2, 3, &c. Join all the points P thus determined with a French curve, until the

FIG. 27.



whole figure A C B C′A is produced. If this figure be accurately drawn, it will be found to consist of two parabolas, demonstrating that the strain upon any point of the flanges of a girder, produced by a uniformly distributed load, varies as the ordinates of a parabola. The simplest way of checking the nature of the curves A C B, A C′C, is to scale the abscissa C 5, and multiply its length by the constant numbers 0·96, 0·84, 0·64, 0·36. The corresponding products should give the lengths of the lines P 4, P 3, P 2, and P 1. If the sectional area, therefore, be constant throughout the girder, the depth will vary and the curve of one of the flanges will be that of a parabola. This is the true form of a bow and string girder, although in actual practice, an arc of a circle is always substituted for the more complicated parabolic outline. Having once determined the strains, the necessary quantity of metal, or the number of square inches in the sectional area of the flanges, is not difficult to determine. All that remains to be done, is to divide the strain by the number of tons that may be safely put upon each square inch of material. As the square inch is the unit of area adopted by all engineers in their calculations, and the ton the unit of weight, the number of tons, that may be safely put upon one square inch, may be termed the unit strain, which for cast iron in tension, is equal to 1·5 tons. Consequently, if the strain at the centre of a girder be 62·5 tons, the number of square inches required in the lower flange will be $\frac{62 \cdot 5}{1 \cdot 5} = 41 \cdot 66$, or, practically, 42 square inches. This quantity must be the "net area" as distinguished from the gross; the term net area signifying the actual area available for resisting the strains, after deducting all rivet and bolt

holes situated in the same line across the breadth of the flange. There is scarcely any necessity for regarding the difference between the net and gross areas in cast-iron girders, as, if properly designed, not more than one $\frac{3}{4}''$ bolt hole should be made, in the same line, across the breadth of the lower flange. Holes in the web are of no consequence, as that portion must be made thicker and stronger than required by absolute theory.

CHAPTER VIII.

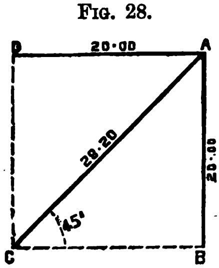
STRAINS UPON THE WEB OF GIRDERS.

THOSE of our readers who have studied the early works relating to the strains upon girders, cannot fail to observe that very scanty information, in many instances none at all, is afforded respecting the strains upon the web. This is undoubtedly, due to the fact that the first investigations were conducted upon solid beams; and it was not until some time after the introduction of wrought iron as a constructive material, and the employment of flanged girders, that this portion of the subject received the attention it deserved. It may be said that it was the lattice, or open web girder, that first compelled engineers, thoroughly to examine the manner, in which the web was affected by strains, since upon an accurate knowledge of this subject, and the correct proportioning of the material, depends the real economical value of this type of girder. In the majority of cases, the sectional area of a plate or solid web must be equal to, and frequently greater than, what theory would demand; and engineers were, consequently, perfectly safe in giving the web a uniform thickness throughout, and simply providing that it was strong enough. It has already been mentioned that in plate and lattice girders, the strains upon the flanges under similar circumstances are identical. If the girder be uniformly loaded, and the depth constant, the breadth of the flange will vary as the ordinates of a parabola. In the first place if transverse strain be the source of all

the strains that can affect a horizontal flanged girder—whether it be supported upon two supports or fixed at one end, as in a cantilever—they may be included under one of three descriptions. The exact manner in which the strains upon a continuous, or plate web, are transmitted from point to point—that is, from the centre to the abutment—has never been clearly understood. It is, however, known that they partake, to some degree, of the nature and direction that have been found to prevail in the case of open web girders. All strains may be ranged under one of three kinds—a vertical, or shearing strain on the web; a horizontal, or strain upon the flanges; and a diagonal strain also upon the web. Before proceeding further, it will be well to consider these a little more in detail, and the mutual relation that exists between them.

In Fig. 28, let A, B represent the vertical or shearing strain, which is always equal to the total weight at any point of a girder, lying between that point and the centre; let A C, represent the diagonal strain, and A D, the horizontal, upon the flanges. From the principle of the "resolution of forces" the diagonal strain A C, may be resolved into the two components A D, and A B, which, in itself, is a circumstance corroborative of the supposition of the manner in which the strains are propagated in a plate or continuous web. Given the value of one of these strains, the other two can readily be determined. For instance, if the value of A B, be 20 tons, all that is required is to plot twenty upon any given scale, draw the line A C, at the assumed angle, 45 degrees on the diagram, and it will measure 28·80 tons, the value of

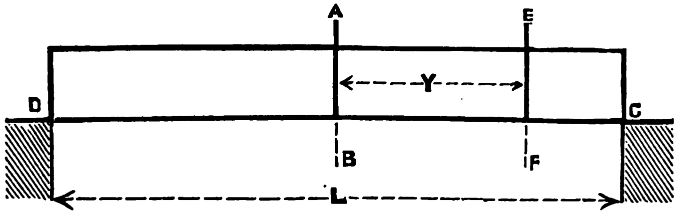
FIG. 28.



the diagonal strain. Completing the parallelogram, the line A D, or C B, will be equal to the horizontal strain upon the flanges. Let W = vertical weight at A, S the diagonal strain upon A C, S^1 that upon A D and θ angle C A D or A C B. Then $S = W \times \text{cosec.}\ \theta$; $S^1 = S \times \text{cosine}\ \theta = W \times \text{cosec.}\ \theta \times \text{cosine}\ \theta = W \times \text{cot.}\ \theta$, when $\theta = 45^0$ cot.; $\theta = 1$ and $S^1 = W$, or the horizontal equals the shearing strain. This method of arriving at the strains, upon the various bars of the web of a lattice girder, will be capable of being strictly followed out, but it is not applicable to the determination of those upon a continuous web. The case of a solid web will be first considered, and then the other principle of construction. Although the strains are supposed to be transferred in a diagonal direction, yet they are assumed to consist simply of a vertical shearing strain, tending to shear the web right through. Whether this be strictly the correct manner of dealing with the subject or not, is of little consequence, as, if the web be made strong enough to withstand the shearing strain, it will also be sufficiently strong to resist it in any other direction. Two general cases will present themselves with respect to the strains upon the web; one in which they result from a uniformly distributed or dead load; and the other from a variable, or rolling weight, frequently called, in contradistinction to the other, a live load. It has been stated that the shearing strain, at any point of a girder, is equal to the total weight situated between that point and the centre of the girder. In Fig. 29, which represents the skeleton elevation of a wrought iron plate girder—the strain upon the web at any part E F, will, therefore, be equal to the total weight distributed over the distance, Y, extending from that point to A B, the centre of the girder. Con-

sequently, if the load per foot run, uniformly distributed over the girder, be one ton, and Y be equal to 10 feet, the

Fig. 29.



shearing strain at E F, will be equal to 10 tons. As the weights with their resulting strains, are transmitted ultimately to the abutments through the means of the web, the shearing strains at those points themselves will be greater than anywhere else, and will equal, as has been previously mentioned, half the total weight distributed over the girder. The shearing strain, therefore, at any point E F, must be less than that at the abutment C, since the weight of that portion of the girder, and the load lying between it and the abutment, have no effect upon it. To find, therefore, the shearing strain at any point of the web, the weight of the part of the girder, and load, situated between it and the nearest abutment, must be deducted from the total weight or shearing strain transmitted to that abutment. Referring to Fig. 29, let the load per foot run, uniformly distributed over the girder, be equal to one ton, let the span equal 30 feet, and let the point E F, be situated 10 feet from the centre line A B. The total pressure at the abutment C, will be equal to $\frac{1 \times 30}{2} = 15$ tons. But in order to obtain the shearing strain at E F, the weight of that portion of the load, situated between it and the abutment C, which

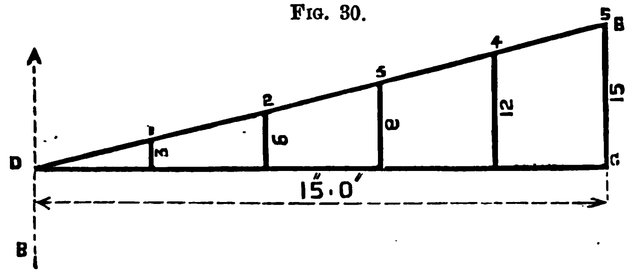
produces no strain upon the former point, must be subtracted. This weight will be equal to the weight per foot run, multiplied by the distance from E F to C, equal to $15 - 10 = 5$. The shearing strain at E F will therefore be equal to $\frac{1 \times 30}{2} - 5 \times 1 = 10$ tons. This proves the former statement, that the shearing strain may be at once obtained by multiplying the weight per foot run, by the distance in feet between the centre of the girder, and the point where the strain is required. Let W = load per foot run uniformly distributed, L = span of girder, then shearing point at any point E F, by assumption equals $W \times Y$, since Y is the distance from centre to E F. To prove this assertion, we have total shearing strain at the nearest abutment $C = \frac{W \times L}{2}$ and weight of load between C and E F $= W\left(\frac{L}{2} - Y\right)$. But

$$\frac{W \times L}{2} - W\left(\frac{L}{2} - Y\right) = W \times Y.$$

It follows from this, that the shearing strain, at any point of the web, is proportional to the distance of that point from the centre, being a maximum at the abutment, and equal to zero at the centre. In all straight girders, similar to that represented in Fig. 29, where the upper and lower flanges are horizontal and parallel, the strains in the web are proportional to the distance Y, but if either of the flanges should be curved, as in a bowstring girder, they no longer obey this law, the curving of either the upper or the lower flange, very considerably modifying the amount and position of the strains. The shearing strains upon the web of the girder, in Fig. 29, vary from zero at the centre to 15 tons at the abutments;

and since they are proportional to Y, they may be geometrically represented by the ordinates of a triangle. Taking half the girder as sufficient for the purpose, make the line C D in Fig. 30, equal to half the span—in the

FIG. 30.



present instance 15 feet. Lay off upon C E, at right angles to C D, upon any given scale, the total pressure transmitted to the abutment C, equal to 15 tons, and draw the line E D, completing the triangle E D C. To find the shearing strains upon the web at every 3 feet apart, plot off the respective distances at the points 1, 2, 3, 4; draw the ordinates, and they will, on being scaled, give the shearing strains at those points. Twenty tons to the inch is the scale to which Fig. 30 is plotted, and the strains are marked thereon.

The investigation of the next case is rather more complicated, but still capable of being fully and clearly explained. Instead of the load being uniformly distributed over the girder, let it be represented by an ordinary railway train, which will successively cover the various portions of the bridge in its passage across. Neglecting the weight of the girder itself, or what amounts to the same, including it in the load, it is evident that, if the rolling load has advanced from the abutment D to the point E F, in Fig. 29, so as to cover the

whole of the larger segment of the girder, into which the line E F divides it, there will be no weight upon that portion of the girder situated between E F and the abutment C. Consequently there will be no weight to be subtracted from that at E F in the calculation, and the shearing strain at that point will equal that transmitted to the abutment C, but it will have a different value to that found for the case of a uniformly distributed load. The shearing strain at E F, when the rolling load is advanced as far as that point, covering the larger segment, will be equal to the weight of the load upon the segment, multiplied by the distance of its centre of gravity from E F. In the present instance, referring to Fig. 29, the calculation will be $\frac{1 \times 25 \times 25}{2 \times 30}$ equal to 10·41 tons. A total weight therefore of 25 tons, causes a greater shearing strain upon the part E F, of the web of a girder, when distributed only over a portion of the girder, namely, over the larger segment, than when it covers the whole span. At first sight this may appear somewhat paradoxical, but those who carefully follow what has been stated respecting the action of the load, and the subtraction of that portion of it situated between the point and the abutment, will have no difficulty in accounting for, and comprehending, the apparent contradiction. To prove this mathematically, let W equal load per foot run on girder, let A equal longer segment, $= \left(\frac{L}{2} + Y\right)$. In Fig. 29 put B equal short segment, $= \frac{L}{2} - Y$. Then shearing strain at

$$\text{E F} = W \times A \times \frac{A}{2 \times L} = \frac{W \times A^2}{2 L}.$$

The strain at E F, when the same load per foot run was uniformly distributed, is $W\frac{(A-B)}{2}$, and the difference is

$$\frac{W A^2}{2(A+B)} - W\frac{(A-B)}{2} = \frac{W}{2}\left\{\frac{A^2}{A+B} - (A-B)\right\} = \frac{W}{2L} \times B^2.$$

The difference between the shearing strains, at the same point under the different conditions, is 0·41 tons, which might have been previously determined, by multiplying the load per foot run by the square of the shorter segment, and dividing the product by twice the span. Thus, $\frac{1 \times 5 \times 5}{2 \times 30} = 0 \cdot 41$ tons. Having once ascertained the shearing strain due to any load uniformly distributed upon any part of the web, it is easy to make the second calculation for the increase due to the rolling load; or, the load being given, the latter may be calculated at once, from the rule given above. Again, if the rolling load be imagined to cover the smaller segment, or the distance between E F and the abutment C, we shall find the shearing strain by calculation to be equal to $\frac{1 \times 5 \times 5}{2 \times 30} = 0 \cdot 41$ tons, or exactly equal to the excess of the strain produced by the rolling load, when it covered the greater segment, over that occasioned by the same load per foot run uniformly distributed.

The maximum strain upon any part E F of the web—taking both the rolling load and the dead weight of the girder and roadway into account—will be, in the present case, assuming the girder and roadway to weigh a quarter of a hundredweight per foot run, found by the following calculation: Total shearing strain at E F, at Fig. 29, is equal to $\frac{1 \times 25 \times 25}{2 \times 30} + 0 \cdot 25 \times 10 = 10 \cdot 41 + 2 \cdot 5$, or,

in round numbers, 13 tons. The longer the segment covered, the greater the shearing strain transmitted to the abutments, where it reaches its maximum, and is equal to half the total weight on the girder. At the centre, with a distributed load, the shearing strain is nothing, since $Y = o$, but with a rolling load, its value is always one-eighth of that load, as may be readily demonstrated. When the load covers only half the girder, the two segments—the loaded and the unloaded—are equal, and moreover, equal to half the span. Under these circumstances, referring to Fig. 29, we shall find, by the rule already given, that the strain will be equal to $\frac{1 \times 15 \times 15}{2 \times 30} = 3 \cdot 75$ tons $= \frac{30}{8}$. The strain at any point due to the rolling load $= \frac{W \times A^2}{2 L}$. At the abutment $A = L$, and strain $= \frac{W}{2} \times L$. At centre $A = \frac{L}{2}$ and strain $= \frac{W \times L}{8}$. These are the practical points of importance relating to the strains upon a solid web or plate girder; and although the action is not so clear as in the bars of a lattice girder, yet the rules are equally true for both descriptions of construction.

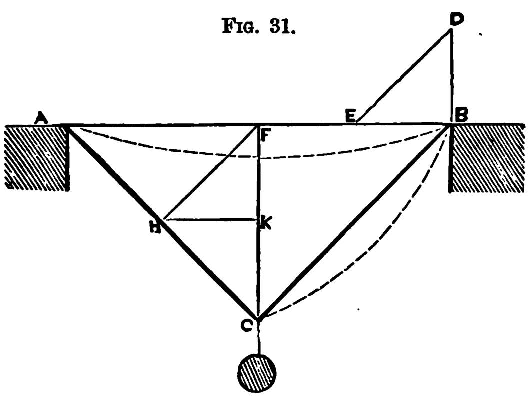
CHAPTER IX.

ELEMENTARY FORMS OF TRUSSES.

THE consideration of the strains upon the web of an open-sided girder, leads to the subject of bracing—a type of construction that was not fully understood and investigated, until subsequent to the extensive adoption of wrought iron by engineers and architects. At first all braced structures were looked upon with suspicion, even by those who, it might be supposed, would have known better. They were virtually condemned by the Commission, appointed to inquire into the application of iron to railway structures, and for a long time were altogether kept in the background by their solid-sided rivals. But as the members of the profession became better educated, as their duties required a larger amount of mathematical and scientific knowledge, than had been previously required, the braced form of girder came gradually into favour, and is now universally admitted to be the type, which permits of the most rigid and accurate determination of both the amount and direction of strains, and of the distribution of the material. The principle of bracing, or of braced structures, may be defined as that which admits of strains being estimated, only in the direction of the length of the separate parts composing the structure. It must not be understood that no part of a system of bracing is ever subjected to a transverse strain, for, in fact, each part must undergo a small strain of this nature, due to the effect of its own weight. But

if the transverse strain become sufficiently great to be included in the calculation, then it interferes with the proper duty of the brace, and the particular advantages of that principle of construction are partially lost. An example will make this clear: The minimum number of parts necessary to constitute a system of bracing is three, and of all figures, the triangle is the best adapted for the purpose, since its form cannot alter without the length of the sides altering also. A perfect system of bracing can be constructed of any polygonal figure, whether it be regular or irregular, symmetrical or unsymmetrical. Let Fig. 31 represent the simplest form

FIG. 31.



of bracing, in which the triangle A B C consists of three bars, A B, B C, and A C, with a weight hung at the point C. The bars being supposed to be connected together by pins at the points A B C, the action of the weight at C will be transferred to the points A and B, and will tend to stretch the bars A C, B C, and to compress A B. The bars A C, B C, are therefore undergoing strain of tension, and are called ties. Upon arriving at the points A and B, the strains tend to

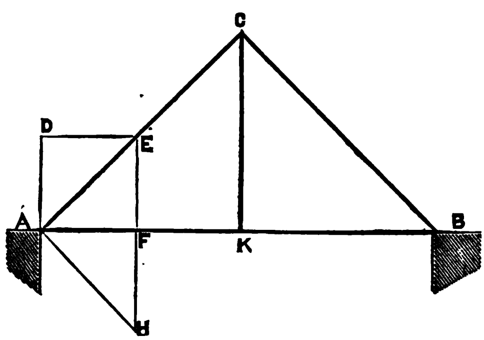
compress or double up the bar A B, which is therefore under a compressive strain, and is termed a strut, in contradistinction to a tie. Strains of tension and compression are usually called strains of opposite character, and are represented in calculation by the signs minus and plus. The actual strain on A B is equal only to that brought upon it by either, and not by both of the bars A C or B C, for one of them acts as a resistance, or fixed point, to the other. The triangle A B C is a complete truss in itself, and the actual strains upon the different members may be readily arrived at. In Fig. 31, let the weight at C equal 10 tons. Upon the principles already laid down, since the point C is situated half-way between the points of reaction A and B, the weight of 10 tons will be conveyed to A and B in equal parts of 5 tons each, and the reaction at each of the points A and B will equal 5 tons. Upon a scale of 10 tons to the inch, plot off 5 tons upon the line B D, perpendicular to A B, that is, make B D equal to 5 tons. Draw D E parallel to the tie B C, and the lines D E, B E will represent respectively the strains upon the ties A C, B C, and the strut A B. The scale upon which the strains are plotted is 10 tons to the inch, and D E equals 7 tons, and B E 5 tons. It has been shown that the strain upon a bar, due to a central weight, $= \frac{W \times L}{4 \times D}$, and whether the weight is placed upon the middle, or hung as in the Fig., the result is the same. In the Fig., W = 10 tons, L = 4 feet, D = 2 feet, and $\frac{W \times L}{4 \times D} = \frac{10 \times 4}{4 \times 2} = 5$ tons.

The strain upon either of the bars $= \frac{W \times L}{4 \times D} \times \text{cosec.}\,\theta$, θ being the triangle between the tie and the horizontal.

These strains are supposed to act on the bars in the direction only of their length, but it is possible that the tie, B C for instance, might be so exceedingly long and heavy as to sag, or deflect, along the dotted line B C. The tie would then be subjected to a transverse strain, which would complicate the calculation very seriously, and, in fact, destroy the principle of the design. Again, the strut A B might either be loaded with a weight at the centre, or be itself so heavy that its own weight might be fairly represented by one placed at its centre, which would induce a transverse strain upon it, and cause it to deflect as shown by the dotted line. Practically, therefore, no part of a truss, which may be regarded as a system of bracing, should ever be subjected to a transverse strain. A brace, whether tie or strut, should act in that capacity, and never be required to undertake the duty of a horizontal girder; in other words, they must always be secured from deflection. This is accomplished in the case of long struts by adopting a sectional area, suitable for resisting buckling, or by introducing subsidiary trusses, which divide the main bars into short lengths, but do not in any way interfere with the continuity, or propagation, of the strain from one end to the other. Referring to Fig. 31, the length of the bar A B evidently affects the question of its being subjected to a transverse strain, and this is an important point to be taken into consideration, when a number of triangles, or single trusses, are joined together, so as to constitute in the aggregate a horizontal girder. The length of the bar, in some measure, determines the limit at which the intersection of the flanges and the web should take place, for if they be too far apart, the supposition of the uniform load, being collected at the apices of the triangles,

will not be correct. Instead of plotting the reaction of the weight at the abutment, upon the line B D, the strains may be found by another graphical operation. From the point C lay off the line C F equal to the whole weight of 10 tons; from F draw F H, and from H draw H K parallel to A B; then F H, or H C, measured upon the same scale, will give the strain upon the tie bars, A C, B C, and H K, that upon the strut A B. If Fig. 31 were to be inverted so that the weight would be placed upon the point C, instead of hanging from it, the amount of the strains would be exactly the same, and would be obtained in precisely the same manner, but would be different in character. The tie bars, A C, B C, would be subjected to compressive strains, and converted into struts, while the strut A B would become a tie. The inversion of Fig. 31, with the introduction of the king rod, C K, would represent the simplest description of roof, consisting of a pair of rafters and a tie beam, and as it is a case frequently occurring, it is shown in Fig. 32. There is, however, a difference in the distri-

FIG. 32.

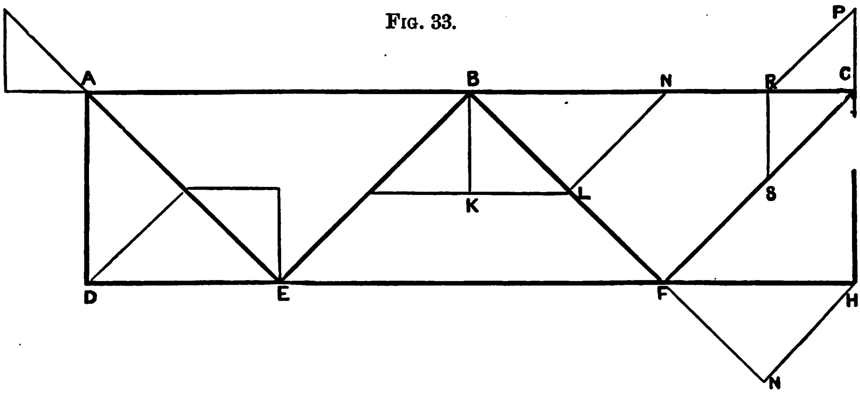


bution of the weight in this last instance. It is not collected at the apex, but supposed to be uniformly dis-

tributed over each rafter, and there will be some slight modification required in calculating the resultant strains. Let there be a total load of 20 tons upon the pair of rafters, or 10 tons upon each; each 10 tons may be considered to act at the centre of gravity, that is, at the centre of the rafter at the point E. Lay off on the same scale as before the line E H = 10 tons; draw H A parallel to C B, then H A will give the compression upon the rafters, and A F the tension on the tie beam A B, or half the weight may be regarded as acting at the point A, and the other half at C, where it is met and resisted by the weight upon the other rafter. Upon this supposition make A D = 5 tons, draw D E parallel to A B; A E and D E will give the strains upon A C, or C B, and upon A B. An analogy manifestly exists between a truss and a horizontal girder, so far as the strains are concerned. It was proved, that a girder would be strained to exactly the same extent, when uniformly loaded with twice the weight, that was suspended from, or placed, on the centre; similarly, the separate parts of the truss in Fig. 32, where the load upon each rafter, or the total load, is twice that placed upon its apex, or suspended as in Fig. 31, are only strained to the same extent; the resulting strains upon the struts are still 7 tons. Where the beam is long, and liable to sag from its own weight, a king rod, C K, is introduced, but with the exception of a slight pull upon it, owing to the tendency of the tie beam to reflect, there is practically no strain at all upon it; it is frequently put in more for the sake of appearances, and to give symmetry to the roof, than for any other reason. The strain upon the struts A C and B C, may be obtained, by multiplying the reaction of the load upon either rafter, by the length of the rafter, and dividing

the product by the rise of the roof. For the strain upon the tie beam, multiply the reaction by the half span, and divide by the rise. The next case constitutes a small girder, and is represented in Fig. 33. First, let there

Fig. 33.



be a weight of 10 tons situated at the apex B, how will it affect the remaining parts of the girder? The horizontal lines are the upper and lower flanges respectively, and the diagonal ones the bars or web; and it will be sufficient to consider one half of the girder. The apex B being at the centre of the girder, the weight will be transferred in equal portions to the two supports A and C, whose reaction will be each 5 tons; consequently, half the weight may be regarded as passing down the bar B F, upon which it will exert a compressive strain. On reaching the point F, it meets with a resistance from the bar F C, and that portion of the flange situated between F, and the centre of the girder. Upon the principle of the resolution of forces, the strain upon the bar B F, or its vertical compound, may be resolved in the direction of F C and E F, and the completion of the parallelogram will give the resulting strains upon those parts of the girder. The action of the compressive strain,

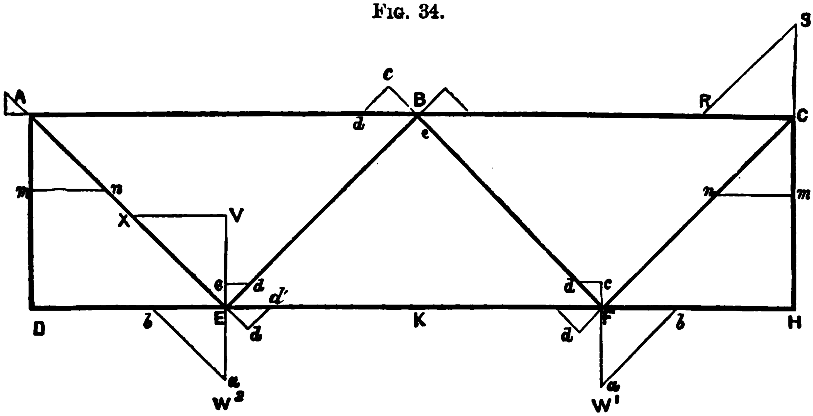
or thrusts, along B F is to lower the point F, and so induce tensile strain on E F and F C. The strain upon E F is met, and counteracted, by one of equivalent amount, on the other side of the centre of the girder, so that it needs no further investigation. The strain, supposed to be travelling along the bar F C, arrives at the point C, pulls upon the pin there, and is met and resisted by the upright bar C H and the flange B C; the former it compresses in a vertical, and the latter in a horizontal direction, towards the centre of the girder. There is no strain upon that portion of the flange lying between F and H, which might be omitted, theoretically speaking. In fact, girders have been constructed with both the parts F H and H C omitted; the Crumlin Viaduct is an example, but there are numerous practical objections against such a mode of construction. The type of girder shown in Fig. 33 is termed a "Warren girder." Its web never consists of more than one series of triangles, that is, it never has any intersection of the bars, and theoretically, it is the type used in the first investigations upon open web girders. Practically, it is a weak form of construction, and has altogether given place to the lattice, which includes two or more series of triangles in its web, with a corresponding number of intersections of the bars. The original "Warren girder" was a combination of cast-iron struts and wrought-iron ties, but owing to several failures having taken place, engineers abandoned the use of cast iron, and employed the latter material only in its construction. Another distinguishing character of the Warren girder is, or rather was, that the connection between the bars and the flanges are made by pins, and not by rivets. Recently, however, this distinction has been entrenched upon, and Warren

girders have been erected, in which rivets have been substituted for pins. The only possible advantage, that can be claimed for the latter method of uniting the flanges and webs is, that it is a rapid one, and can be performed in a foreign country without the necessity of employing skilled labour. For this reason Warren girders have been extensively employed in India, and elsewhere abroad, but it is questionable if the reason is a very valid one. It would be just as easy to rig up a small portable forge, and drive a few rivets, as to insert a pin, and there is no comparison between the rigidity and durability of the two methods. Having investigated the manner in which the weight at B in Fig. 33 is supposed to act, the amount of the various strains it gives rise to, on the different members of the girder, can be ascertained. It is not to be understood that the strains actually behave in the manner mentioned, for very little is really known about the matter, but to those not thoroughly acquainted with the subject, a familiar and easily comprehended style of explanation will be of use. Since half the weight at B is transferred to each abutment, lay off in the diagram, upon a scale of 10 tons to the inch, B K = 5 tons, K L parallel to the top flange, and L N parallel to B E, then B L, measured on the same scale, will give the thrust or compressive strain on B F, and B N the horizontal strain on E F. This strain upon the top or bottom flange being transferred to the point F, lay off F N = B L = the strain upon B F, and draw N H parallel to F C, N H, and F H = B N, represent the tensile strains upon the tie F C, and upon the bottom flange between the centre of the girder and the point F. Transferring the vertical component to the point C, and plotting it on the line C P, drawing the line P R parallel

to F C, give R C a second compressive strain upon B C, while that on C H equals C P, which is the vertical reaction of the support. The same results may also be arrived at by plotting the actual strain upon the bar F C, instead of its vertical component, care being taken in all cases that the resultants are parallel to the direction of the bars upon which the strains are required. A number of different ways of arriving at the strains will always present themselves to those who really study and investigate the question. Other methods are shown to the left of the centre, and as the diagram is drawn to scale, the reader can easily verify the different lines, and the value of the strains. If the weight of 10 tons were uniformly distributed over the top of the girder in Fig. 33, the strains would be just half what they have been found to be, which has been shown to be a universal rule. This would follow from the alteration in the distribution of the load. Instead of having the whole 10 tons upon the centre at B, there would be 5 tons at B, and $2\frac{1}{2}$ respectively at A and C, which would only produce strains at A D and C H, and which, added to that they would receive as the vertical resultant of the strains upon the diagonals A E and F C, would equal 5 tons as before. In order to explain the action of a couple of weights, let W^1 and W^2, each equal to 5 tons, be suspended from the points E and F in Fig. 34. To find the strains upon the separate parts, let the effect of W^1 be first considered. Upon the principle of the lever, and from the diagram, one quarter of its weight is transferred to the abutment A, and three quarters to C. Make F $a = 3 \cdot 75$ tons, draw a b parallel to F C, then a b measured on a scale of 10 tons to the inch, will give the tensile strain on F C. This strain is again transferred to the point C. Plot its

vertical component F a = C m, and draw $m\ n$ parallel to the top flange, $m\ n$ equals the compressive strain on B C,

Fig. 34.



and C m that on C H. That portion of W^1 which is conveyed to the support C being now accounted for, it remains to ascertain what becomes of the other portion. First, it is well clearly to state the effect of W^1 upon those parts of the girder situated between it and the abutment C. It has been shown that W^1 causes a strain of $-$ 5·25 tons on the bar F C, of + 3·75 tons upon B C, and of + 3·75 upon the upright pillar C H. The vertical component of the remaining portion is 1·25 tons, which is equal to F e, and d F equals the tensile strain upon B F, which is transferred to B, and there creates a similar compressive strain, B c, of equal amount upon B E, and a horizontal strain upon the pin B, which is balanced by one from the opposite side. This strain is again conveyed to the bar A E, as shown by E d in the diagram, where it causes a tensile strain equal to E d upon A E, and on K E equal to $e\ d'$, and finally pulls upon A, where it causes a compressive strain upon A B and on A D. The weight W^1 therefore causes a strain of $-$ 1·75 tons upon the bar F B, of + 1·75 tons upon

B E, of − 1·75 upon A E, of − 2·50 tons on K E, of + 1·72 tons on A B, and of 1·25 tons upon A D. By similar reasoning, the weight W^2 causes a strain of − 5·25 tons upon A E, of + 3·75 tons upon A B and A D, of − 1·75 tons upon B E, of + 1·75 tons on B F, of − 1·75 tons on F C, and of + 1·25 tons upon B C and C H. No part of a girder can undergo tensile and compressive strain at one and the same time; therefore, in summing up the actual strain upon any bar, the algebraical sum of the separate strains must be taken: Thus, if a bar suffer a tensile strain of 10 tons, and also is subjected to a thrust of 5 tons, the actual strain upon it is 5—tons.

It will be sufficient to sum up the strains for one-half the girder in Fig. 34, as the other half will be similarly circumstanced. The upright C H is subjected to a compressive strain of 3·75 tons from W^1, and of 1·25 tons from W^2, therefore total strain upon C H = + 5 tons. The portion of the flange B C is subjected to a strain of + 3·75 tons from W^1, and + 1·25 tons from W^2; total strain equals + 5 tons also. This follows from what has been previously stated, that when the angle between the diagonal bar and the flanges is 45°, the horizontal equals the vertical strains, or the shearing strains equal those upon the flanges. As the bars B F, B E are subjected to a compressive strain of 1·75 tons, and also a tensile strain of the same amount, the actual strain upon them is equal to zero, and this proves one of the general rules relating to lattice girders, namely, that with a uniformly distributed dead load, the strain upon the central bars is equal to zero. The total strain upon the top flange in Fig. 34 is 5 tons, and that upon the bars A E, F C 7 tons. The total strain upon the lower flange will equal 6·25

tons, which is rather more than that found for the upper one. It follows from Fig. 33 that when a load is placed at the centre of a girder, the strain upon all the diagonal bars is the same, and is equal to half of the weight, multiplied by the cosecant of the angle of inclination between the diagonal and the horizontal. In the example in Fig. 33, the angle of inclination of the bars to the horizon is $45°$, and the natural cosecant $1 \cdot 414$. Multiplying half the weight at B gives $5 \times 1 \cdot 414 = 7 \cdot 070$, or practically 7 tons, as the strain upon the bars B F, B E, F C, A E. Similarly, the strain upon the flange at C, equals the vertical reaction multiplied by the cotangent of the same angle $= 5 \times 1 \cdot 000 = 5$ tons. The following general rules always hold with lattice girders:—With a load placed at the centre, the strains upon all the diagonal bars are equal, and the strains upon the flanges increase from the ends towards the centre, where they are equal to the total weight, multiplied by the span, and divided by four times the depth of the girder. With a load uniformly distributed, the strains upon the diagonal bars are equal to zero at the centre, and increase towards the ends; the strain upon the last bar, when there is only one system of triangles, being always equal to half the total load upon the girder, multiplied by the cosecant of the angle of inclination of the bars to the horizontal. The strains upon the flanges always increase from the ends towards the centre, where they are always equal to the total weight, multiplied by the span, and divided by eight times the depth. Having ascertained that there is no strain upon the central bars of a girder subjected to a uniformly distributed dead load, the shortest way to find the strains upon the truss in Fig. 34 would be to lay off E V equal

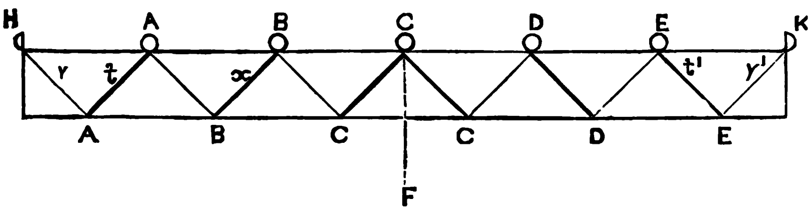
to the weight of 5 tons, draw V X parallel to the bottom flange, then E V measures the vertical strain upon the end pillar A D; V X, that on the top flange, and X E that upon the bars A E and F C. All strains upon the upper flange are of a compressive nature, and all upon the lower, of a tensile. The upper flange tends to become shortened, and the lower lengthened. In these two examples the weight is at the centre in the first, and uniformly distributed in the second; consequently, the strains upon the flanges in the latter case are known to be only half of those in the former instance, and are respectively 5 and 10 tons. When the load is uniformly distributed, it is only necessary, in determining the strains, to consider one-half of a trussed girder. This may be deduced from the example of the two weights. The tensile strain brought upon either of the central bars, by the weights situated upon one side of the centre, is balanced by the compressive strain, induced by the weight placed upon the other side, so that they may both be neglected. It is therefore sufficient to take the whole load placed at any apex, and consider its action upon the bars, and those portions of the flanges situated between it and the nearest abutment, and neglect the action upon the parts between it and the centre, as the strains will be balanced by others of an opposite nature, from the weights at the other side of the centre of the girder.

CHAPTER X.

EFFECT OF THE POSITION OF THE LOAD.

To sum up the algebraical sum of the strains brought upon each bar by the weights, considered as uniformly distributed over the girder, would be a tedious and, at the same time, a very unnecessary task. Let Fig. 35

FIG. 35.



represent a girder, uniformly loaded throughout its length, or, what amounts to the same, suppose the weights collected at the several apices of the triangles. Selecting any bar x, the total strain upon it will be equal to the shearing strain at the apex B, multiplied by the cosecant of the angle of the inclination of the bar to the horizon. The shearing strain will be equal, as already stated, to the sum of the weights situated between the apex and the centre of the girder. If each of the weights A, B, C, &c., be equal to 1 ton, then the total strain upon the bar x equals $1 \cdot 5 \times 1 \cdot 4 = 2 \cdot 10$ tons, assuming the angle of inclination of the bar to be 45°. Let us now proceed to obtain this result by considering the action of each individual weight. Commencing with

the weight of 1 ton at E, upon the principle of the lever, five-sixths of it are transferred to the abutment K, and one-sixth to H. The vertical component of the strain brought upon the bar x, by the weight placed at E, is a compression of one-sixth of a ton. Similarly, the vertical component of strain upon x, due to the weight D, will be two-sixths of a ton, that of the weight C three-sixths of a ton, and that of B four-sixths of a ton. All these strains are compressive, and summing up we find the total to be equal to $(\frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6}) = \frac{10}{6} = 1\frac{4}{6}$ ton. But the bar x is also subject to a tensile strain from the effect of the weight A, which is equal vertically to one-sixth of a ton, so that the total strain upon the bar x is equal $1\frac{4}{6} - \frac{1}{6} = 1\frac{1}{2}$ ton as before. Multiplying this by $1 \cdot 4$, we obtain the result to be as before $2 \cdot 10$ tons.

A little reflection will point out that the tensile strain, brought by the weight at A upon x, is neutralized by the compressive strain resulting from the action of that at E, and that the compressive strain coming from the weight at D, is of exactly the same amount as that portion of the weight at B, which does not pass down x towards H. The reason why the last bars, or those nearest the abutments, are always strained to a maximum is thus apparent. Since the weights at K and H, which are respectively equal to half those at the apices of the triangles, cause no strain upon the bars y, y', but are supported altogether by the vertical reaction of the abutment, there is no neutralizing strain upon them. The bar y is strained in tension by all the weights A, B, C, D, E, and there is no compressive strain at H, to be subtracted from their united action. In the present instance the bars t, t', are also strained to a maximum in compression, being evidently affected by the same

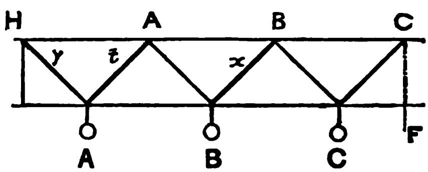
weights to the same extent as y and y', but the strain is of a compressive instead of a tensile character. The bars y and t and y' and t' are said to be pairs, that is, they are acted upon by strains of the same amount, but of an opposite nature. From the rules previously laid down, the strain upon t and t' is a compression of half the total load upon the apices of the triangles, multiplied by 1·4, and that upon y and y' a tension of the same amount. Both these strains consequently equal $(2 \cdot 5 \times 1 \cdot 4) = 3 \cdot 5$ tons. The vertical pressure upon each of the uprights H and K, will be equal to half the total load upon the girder, equal to 3 tons.

Having now made perfectly clear the manner in which the bars are affected by the several weights, the strains upon the flanges have next to be considered. These are *cumulative*, that is, the total strain upon the centre portion B C is equal to the sum of its own proper strain, and those upon the other portions A B, A H. Similarly, the total strain upon C D, equals strain upon C D, plus that on D E. The strain upon the whole of the upper flange is compressive, and is induced by the bars tending to compress or double it up towards the centre. That upon the lower flange is tensile in character, and its tendency is to stretch the flange from the centre. The total tensile strain upon C C, is equal to its own, plus that upon D C, plus that upon D E, or equal to its own, plus that upon B C, plus that upon A B. It will be seen hereafter, that the strains upon the upper and lower flanges are not identically, although very nearly equal, and, moreover, the actual amount of each will depend in a great measure, on the position of the load, whether it be placed at the top or the bottom of the girder.

That the strains upon the various bars are also affected

by the position of the load, a little consideration will serve to show. As it has been proved that it is only necessary to regard one-half of a girder, let Fig. 36 represent one-half of Fig. 35, and let the weights be situated as represented in the latter figure, the same letters being used for both. Since it is necessary to consider the action of the weights, upon only those bars that are placed between it and the nearest abutment, there is, therefore, no strain whatever upon the bar C C, when the weights are situated at the lower apices of the triangles. When they were placed upon the top, the bar C C was subjected to a compressive strain of 0·7 tons, but in the present instance it is free from strain. The reason of this is at once apparent, if we imagine the other weight to be placed at C, upon the other side of the centre line C F, in Fig. 36, as was explained in the last chapter. With a weight of 1 ton at C, in Fig. 36, the tensile strain upon the bar C B will be 1·4 tons, and a similar compressive strain will be exerted on B B and A A, also a tensile one of the same amount upon B A and A H. These strains are those produced on the bars by the action of the weight at C, which is thus accounted for. The weight placed at B will exert corresponding strains of the same amount upon the bars that are situated between it and the abutment, that is upon B A, A A, and A H. The bar A H will finally receive a third strain due to the action of the weight at A, and the sums of the strains upon the various bars will be as follows:

FIG. 36.



C C = $0 \cdot 0$; C B = $-1 \cdot 4$; B B = $+1 \cdot 4$; B A = $-1 \cdot 4 - 1 \cdot 4 = -2 \cdot 80$; A A = $+1 \cdot 4 + 1 \cdot 4 = 2 \cdot 80$; and A H = $-1 \cdot 4 - 1 \cdot 4 - 1 \cdot 4 = -4 \cdot 2$ tons.

In the first place, it is to be observed that the arrangement among the bars themselves is changed by altering the position of the load. Those which were pairs in the former instance are no longer so now. The pairs, when the weights were at top, were C C and C B; B B and B A; A A and A H. Now they are C B and B B; B A and A A. It will also be noticed that there is a greater total load upon the girder by the arrangement adopted in Fig. 36. If we take the half girder in Fig. 35, the whole load upon it is $2\frac{1}{2}$ tons, since half a ton is supported directly by the vertical reaction of the abutment at H, whereas in Fig. 36 the whole 3 tons is supported at the lower points of the triangles. Consequently the strain upon the end bar Y will be greater than in the other case. In Fig. 35 it was shown to be equal to $2 \cdot 5 \times 1 \cdot 4 = 3 \cdot 5$ tons. By the same rule it will now be equal to $3 + 1 \cdot 2 = 4 \cdot 2$ tons, as found about by summation. The difference is evidently the diagonal component of the vertical load of half a ton, which is not carried by the support as in Fig. 35, and which is equal to $0 \cdot 5 \times 1 \cdot 4 = 0 \cdot 7$ tons. The vertical pressure upon the upright at H will not undergo any change, but be 3 tons, as in the other example. The strain upon the upper flanges will be also slightly increased by this arrangement of the weights, but this will be considered more in detail in a succeeding example.

CHAPTER XI.

APPLICATION OF THE GRAPHIC METHOD.

In Fig. 37 is represented the skeleton elevation of half of a girder having a span of 80′. The bracing is arranged in equilateral triangles, and the load is at the rate of half a ton per running foot, or 40 tons, uniformly distributed over the girder. Since the triangles are equilateral, the depth of the girder may be easily calculated. The length of any diagonal bar A H is equal to the length of one bay, or portion of the flange A B, situated between any two apices of a triangle. This, by the figure, is equal to 10′; so that, calling D the depth, we have $D = \sqrt{(100-25)} = 8 \cdot 66'$. This value for the depth will be required in checking the strains upon the flanges, ascertained by scaling the corresponding lines in the diagram. In the first instance, let us suppose the load to be uniformly distributed over the top of the girder. It will be arranged as follows:—There will be a weight of 5 tons upon each of the apices B, C, D, E, and one of $2\frac{1}{2}$ tons upon A. Regarding only one-half of the girder, there will be only half the weight at the central apex E, which will have to be taken into account. Consequently, the distribution of the separate weights upon the girder will be $2\frac{1}{2}$ tons upon A, 5 tons upon B, 5 tons upon C, 5 tons upon D, and $2\frac{1}{2}$ tons upon E, making in all 20 tons, or half the total load. In Fig. 37, the skeleton elevation of the girder is drawn to a scale of 8′ to the inch, and the strains plotted to a scale

of 4 tons to the inch, which allows of an estimation of one-tenth with facility, so that the strains can all be accurately scaled off.

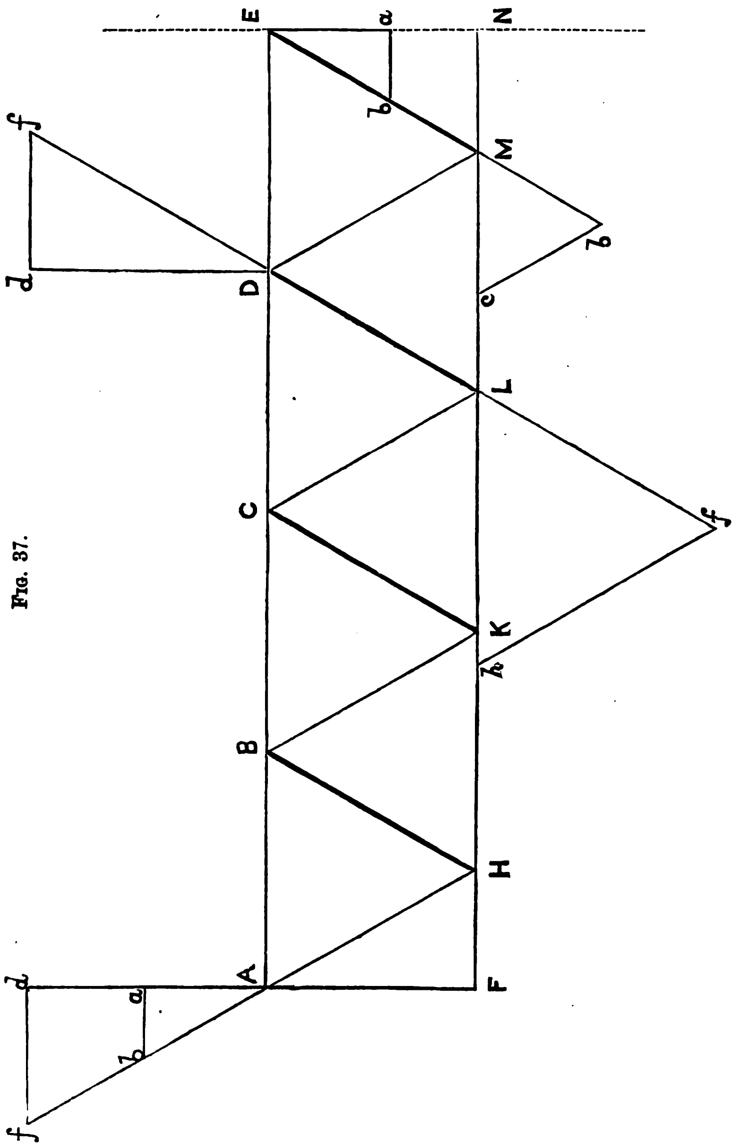


FIG. 37.

Commencing at the centre, the weight of 2·5 tons which passes down the bar E M, causing a compressive strain upon it from E to M, where it meets with an equal and opposite reaction. The strains are of course all ultimately transferred to the upright A F, as will be subsequently explained. To determine the effect of the first weight of 2·5 tons at the central apex E, plot off upon a scale of 4 tons to the inch the line E a = 2·5 tons; draw a b parallel to the top flange, then E b represents the strain upon the bar E M = 2·9 tons; while a b gives the thrust against the central pin E, which is met and counteracted by one upon the other half of the girder, and may therefore be disregarded. This strain of 2·9 tons—confining the attention for the moment to the diagonal bars—is induced in all of them, subjecting the bar D M to a strain of − 2·9 tons; D L to a strain of + 2·9 tons; C L to a strain of − 2·9 tons; C K and B H to compressive strains of the same amount, and B K, A H, to tensile strains of − 2·9 tons. Upon arriving at the point A the strain of 2·9 tons is evidently taken partly by the action of the flange A B, and partly by that of the pillar or upright A F. Omitting the consideration of the first for the present, upon the prolongation of H A, lay off A b = 2·9 tons, draw b a parallel to the top flange of the girder, and prolong F A to meet it in a. The compressive strain upon the pillar equals A a = 2·5 tons, or the vertical component of the strains upon the diagonal. We thus perceive that the original weight of 2·5 tons is transferred to the abutment unchanged in amount, notwithstanding the various members of the structure through which it has to pass, and by which it is conducted, so to speak, to its final resting place. It has been already shown that the strains upon the dia-

gonal bars are all ultimately referred to the abutment or point of reaction, and that their amount increases with the distance that each bar is from the centre, being a minimum at the latter point, and a maximum at the former. In the same manner the strains upon the flanges are all referred to the centre of the girder, where they attain a maximum, being a minimum at the support or abutment.

Having disposed of the action of the weight of 2·5 tons placed at the central apex E of the girder, in a diagonal and vertical direction, there yet remains its horizontal component to be accounted for, or the strains it induces upon the flanges. To find the amount of these, lay off M b = E b, upon the prolongation of the diagonal E M, draw b c parallel to M D, to meet the lower flange; then c M gives the tensile strain upon the portion of the lower flange M N, and the compressive strain upon the part D E of the upper flange. By the construction, this strain is evidently equal to that upon the bars, or equal to 2·9 tons. The effect of the weight 2·5 tons placed at E, upon the flanges is as follows:—It causes a separate tensile strain of 2·9 tons upon M N, L M, K L, and H K; and, from what has been previously stated, the total strain resulting from this one weight upon M N equals $2 \cdot 9 \times 4 = 11 \cdot 6$ tons. Similarly the total strain upon L M = $2 \cdot 9 \times 3 = 8 \cdot 7$ tons, that upon K L = $2 \cdot 9 \times 2 = 5 \cdot 8$ tons, and upon H K = $2 \cdot 9 \times 1 = 2 \cdot 9$ tons. The total strain upon M N is, therefore, equal to its own strain, plus those upon the other portions of the flange lying between it and the point of support, thus proving the statement made, when explaining Figs. 35 and 36.

Let us now consider the upper flange, and here we shall find some difference. The strain upon D E will,

by analogous reasoning, be evidently equal to its own strain, plus those upon the remaining parts of the flange; and, at first sight, it might appear that the total would correspond with that already found for the lower flange, but a little consideration will show the fallacy of this assumption. Upon the lower flange the successive strains are all equal to M c, but this is not the case in the upper. They are all equal to M c, with the exception of that upon the last part A B, which is not equal to M c, but to a b. The method of finding a b has already been demonstrated, and its value will be seen, upon measuring it with the scale of 4 tons to the inch, to be just one-half of M c, that is equal to 1·45 tons. The strains upon the upper flange from the action of the weight of 2·5 tons at E will manifestly be as follows:—They are all compressive, and that upon D E will be equal to $2 \cdot 9 \times 3 \cdot 5 = 10 \cdot 15$ tons; and that upon C D $= 2 \cdot 9 \times 2 \cdot 5 = 7 \cdot 25$ tons; that upon B C $= 2 \cdot 9 \times 1 \cdot 5 = 4 \cdot 35$ tons, and that upon A B $= 2 \cdot 9 \times 0 \cdot 5 = 1 \cdot 45$ tons. We have thus accounted for the action of the weight of 2·5 tons at the central apex E, throughout the whole girder, diagonally, vertically, and horizontally, and it remains now to determine the effect of the others.

It will be sufficient for the purpose to trace throughout the effect of the weight at D, since the action of the others at the other apices will be similar. The weight at D is 5 tons, or just double that of E, and it may, therefore, be anticipated that its effect upon the various members of the girders will be double that produced by the weight E. So far as the diagonal bars are concerned, this surmise is correct, but it does not hold strictly for the flanges, as will presently be perceived. To consider the diagonal bars first, plot off upon the same scale as

before, D d = 5 tons; draw $d\ f$ to meet the bar L D produced. Then D F will be found on scaling to measure 5·8 tons, and will equal the alternate compressive and tensile strain upon all the bars from D L to H A. The diagonal bars undergoing a compressive strain, or the struts, are shown in Fig. 37, by the thick lines, while the ties, or those strained in tension, are represented by the thin lines. It is readily perceivable, that a weight of 5 tons upon C will induce strains of similar amount upon the bars C K, K B, B H, and H A, and that the weight upon B, will affect to the same extent, the bars B H and H A. The pillar A F will support the vertical component of these separate strains; that is, the actual amount of the weight itself. This may be proved by producing H A to f, making A F = the strain upon the bar = 5·8 tons, and drawing $f\ d$ to meet the pillar produced in d; then A d will measure 5 tons, the vertical compression upon the pillar. We may now sum up the strains upon the several bars in the accompanying Table, and, to show the accuracy of the graphic method, check the sums by mathematical calculation.

TABLE I.

Weights at	Bars.								
	EM.	DM.	DL.	CL.	CK.	BK.	BH.	AH.	AF.
E	+2·9	−2·9	+2·9	−2·9	+ 2·9	− 2·9	+ 2·9	− 2·9	+ 2·5
D	..	..	+5·8	−5·8	+ 5·8	− 5·8	+ 5·8	− 5·8	+ 5·0
C	..	..	..	..	+ 5·8	− 5·8	+ 5·8	− 5·8	+ 5·0
B	..	..	..	..	..	..	+ 5·8	− 5·8	+ 5·0
	+2·9	−2·9	+8·7	−8·7	+14·5	−14·5	+20·3	−20·3	+17·5

Let us now proceed to ascertain the accuracy of the results in the Table. The strain upon any bar is equal to the total weights, situated between it, and the centre

of the girder, multiplied by the cosecant of the angle of inclination of the bars. Take the bar C K or B K, since they are pairs, and consequently the strains upon them will be equal in amount, although opposite in nature. The result, by the Table, is $\pm$ 14·5 tons. By calculation it is equal to $12 \cdot 5 \times 1 \cdot 1547 = 14 \cdot 43$ tons, an approximation quite sufficiently accurate for all practical purposes. Similar accurate results will attend the calculation of the strains upon the others. The strain upon the vertical bar A F is known to be equal to half the total weight supported by the girder $= \frac{35}{2} = 17 \cdot 5$ tons, as in the Table. The strains upon the diagonals having been fully explained, both in nature and amount, those upon the flanges now claim attention; and it will be sufficient to inquire minutely only into the action of the weight at D, as that of the others will be similar. Owing to the reaction of the weight at D, the bar D L will push against the portion of the upper flange D E, and compress it towards E. The amount of this strain is given by $d\,f$, the method of finding which has been already explained when treating of the strains upon the bars. On scaling $d\,f$, it will measure 2·9 tons. A little consideration will point out, that the weights at C and B will also push C D and B C with strains of the same amount; and, as these strains are all referred to the centre, the portion of the upper flange D E will be pushed or compressed by the weights at D, C, and B to the extent of $2 \cdot 9 \times 3$ or 8·7 tons.

This may be termed the direct action of the weights at these apices, and we have now to consider the additional strains brought upon the top, as well as upon the bottom, flange by the action of the bars. The weight at

D being ultimately transferred to the abutment, will alternately stretch and compress both bars and flanges on its way. The strain upon the bar D L has been proved to be equal to $5 \cdot 8$ tons. Lay off upon the bar D L produced, $Lf = DF = 5 \cdot 8$ tons, and draw $f\,h$ to meet the lower flange; then L h, measured on the same scale, will equal the additional strain brought upon each separate portion of the flanges, with the exception of that upon the end portion A B. The strain upon this part, due to the action of the weight at the several apices, will be only half that brought upon the other portions of the flanges, as may be seen by measuring $f\,d$, already plotted, in order to obtain the vertical strain upon the pillar A F. Consequently the total additional strain upon D E, owing to the weight upon D, will be equal to $2 \cdot 9$ tons $+ (5 \cdot 8 \times 2 \cdot 5) = 17 \cdot 40$ tons. Similarly the total strain due from the weight at C will equal $2 \cdot 9 + (5 \cdot 8 \times 1 \cdot 5) = 11 \cdot 60$ tons; and that from the weight at B $= 2 \cdot 9 + (5 \cdot 8 \times 0 \cdot 5) = 5 \cdot 8$ tons. From above the strain upon D E was found to be equal to $2 \cdot 9 \times 3 \cdot 5 = 10 \cdot 15$ tons, and adding them together, we obtain the total strain upon the central portion to be equal to $44 \cdot 95$ tons. The strain upon C D will be manifestly equal to that upon D E, less the push of the pin D, and less the pull upon the bar M D, that is, less the value of these compressive strains on D E, that is equal $44 \cdot 95 - (2 \cdot 9 + 2 \cdot 9) = 39 \cdot 15$ tons, and so on for the other portions of the flanges. These strains may also be obtained by the process of summation, as shown in the accompanying Table of the strains upon the flanges.

In a similar manner the strains upon the separate portion of the lower flange may be determined. The strain upon the part M N, resulting from the action of

the weight at D, will be equal to three times the value of L h = (5·8 × 3) = 17·4 tons. The weight at C will produce a strain of (5·8 × 2) = 11·6 tons; and that at B of (5·8 × 1) = 5·8 tons. The strain upon M N from the weight at E was shown to be equal to 2·9 × 4 = 11·6 tons; and, summing up, the total strain will be found to be equal 46·4 tons. Subtracting the strain upon the top flange from that on the bottom, we have a difference of 1·45 tons; so that, practically, the statement made at the commencement, that the strains upon both flanges were equal holds good. The strain upon L M is equal to that upon M N, minus the compressive strain, resolved horizontally, of the bar E M, equal to 46·4 − 2·9 = 43·5 tons, and so on for the remaining portions of the flanges. In the following Table the strains upon the flanges are summed up, and our younger readers should carefully consider them:—

Table II.

Weights at	Parts of the Flanges.								
	AB.	BC.	CD.	DE.	FH.	HK.	KL.	LM.	MN.
E	+ 1·5	+ 1·5	+ 1·5	+ 1·5	0·0	− 2·9	− 2·9	− 2·9	− 2·9
	+ 0·0	+ 2·9	+ 2·9	+ 2·9	0·0	− 0·0	− 2·9	− 2·9	− 2·9
	+ 0·0	+ 0·0	+ 2·9	+ 2·9	0·0	− 0·0	− 0·0	− 2·9	− 2·9
	+ 0·0	+ 0·0	+ 0·0	+ 2·9	0·0	− 0·0	− 0·0	− 0·0	− 2·9
D	+ 2·9	+ 2·9	+ 2·9	+ 2·9	0·0	− 5·8	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 5·8	+ 5·8	+ 5·8	0·0	− 0·0	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 0·0	+ 5·8	+ 5·8	0·0	− 0·0	− 0·0	− 5·8	− 5·8
	+ 0·0	+ 0·0	+ 0·0	+ 2·9	0·0	− 0·0	− 0·0	− 0·0	− 0·0
C	+ 2·9	+ 2·9	+ 2·9	+ 2·9	0·0	− 5·8	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 5·8	+ 5·8	+ 5·8	0·0	− 0·0	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 0·0	+ 2·9	+ 2·9	0·0	− 0·0	− 0·0	− 0·0	− 0·0
B	+ 2·9	+ 2·9	+ 2·9	+ 2·9	0·0	− 5·8	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 2·9	+ 2·9	+ 2·9	0·0	− 0·0	− 0·0	− 0·0	− 0·0
Total ..	+10·2	+27·6	+39·2	+45·0	0·0	−20·3	−34·8	−43·5	−46·4

It is to be noticed that in Table II. the total strain for the central part D E of the upper flange is 45·0 instead

of 44·95. This trifling difference is due to making the value of the line *a b* in Fig. 37, equal to 1·5 instead of 1·45, in order to save a second place of decimals in the Table, which with the exception of that one strain would be rows of ciphers.

The strains upon the whole girder, have now been investigated and determined, with an accuracy equal to mathematical calculation, and quite sufficient to prove, that with proper care and attention to the drawing of the diagram, and the plotting of the strains, this method is susceptible of the greatest precision. There still remains the case of a passing load to be considered, but those who have made themselves masters of the rules already elucidated, will have no difficulty in following up the subject.

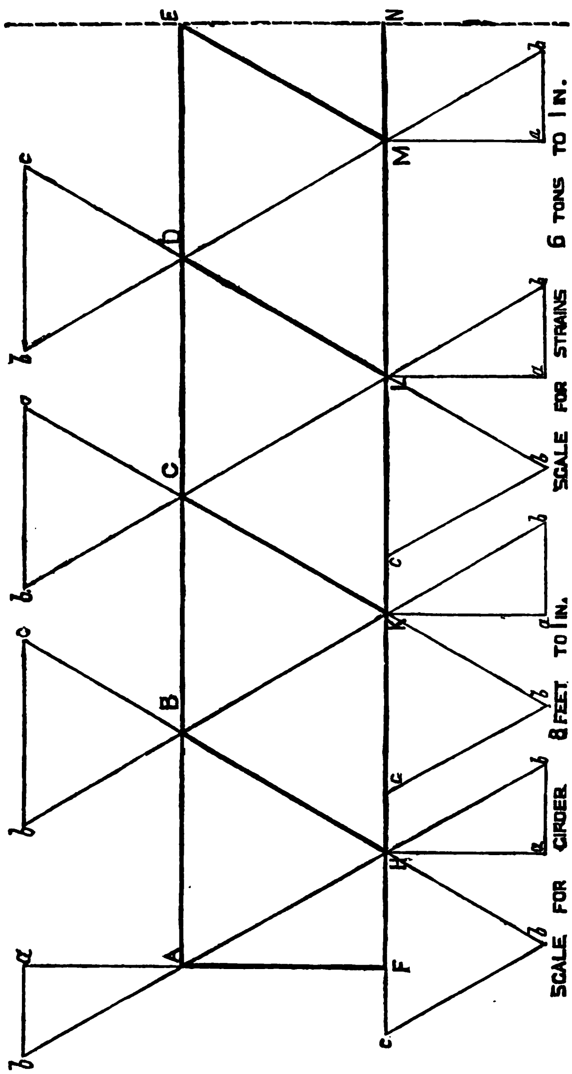
CHAPTER XII.

EFFECT OF THE POSITION OF THE LOAD—CONTINUED.

Having investigated graphically and analytically in the last two chapters, the case of a girder uniformly loaded upon the top flange, we will now consider what difference is caused in the strains upon the various members, by placing the load upon the lower flange. The same example is selected for the express purpose of affording a ready method of comparison at a glance, and also because instances occur in which the beginner, although able to take out the strains upon a structure when the load is distributed in one particular manner, is completely at a loss how to proceed when it is arranged in a different manner. Referring to Fig. 38, there is a load of 5 tons upon each of the apices M, L, K, and H, or 20 tons in all, which is an increase of 2·5 tons more than the total weight upon the girder, when the load was uniformly distributed over the upper flange. In that arrangement 2·5 tons was directly supported by the pillar A F, and caused no strain upon the remaining members of the girder. Commencing with the diagonal bars, and taking the weight placed at M, there are strains of tension induced upon M D and M E. That upon M E is balanced by a compressive strain of the same amount, from the weight of 5 tons placed at the corresponding apex at the other side of the centre line E N, and the two strains neutralize one another, the resultant being equal to zero. This result might also have been

anticipated from what has been already stated, that in determining the strains upon the half girder, it was

Fig. 38.



necessary only to have regard to the bars situated between the weight and the point of support. Con-

sequently, upon this assumption, the strain upon M E is equal to cipher.

The process of reasoning is analogous to that employed in the previous case: To find the strains upon all the bars produced by the action of the weight of 5 tons at M, make M a = 5 tons, draw a b parallel to the bottom flange, to meet the bar D M, produced to b. Then M b, measured upon the same scale, will give the strain upon all the diagonal bars situated between M and the support at F. This strain will be found equal to 5·8 tons, and will be minus or plus, that is, tensile or compressive, according as the bars are ties or struts, the latter being shown in the diagram by thick lines. The strain upon A F will not be equal to 5·8 tons, but to 5 tons, as may be seen from the figure, by plotting off upon the bar A H produced, A b = its own strain = 5·8 tons, and drawing b a parallel to the top flange, to meet F A produced, A a = 5 tons. Similarly the weight of 5 tons at L, induces alternate strains of tension and compression of the same amount, upon all the diagonal bars between it and the support, and the same effect is produced by the respective weights at K and H. The sum of the strains gives the total strain upon each bar. It is worth noticing here the different manner, in fact, the completely opposite manner, in which the process of summation of the strains upon the bars and flanges is conducted. In the latter they sum up, as it were, from the end of the girder, in the former from the centre. Thus the strain upon M D is equal to M b = 5·8 tons; that upon L C = M b + L b = 2 × 5·8 = 11·6 tons; that upon K B = M b + L b + K b, and so on for those upon the remaining bars H A. In the flanges, for instance, the strain upon D E will equal its

own strain + that upon A B + that upon B C + that upon C D; the summation proceeding in completely the opposite direction. In Table III. are tabulated the results of the action of the different weights upon all the bars, and if it be compared with the diagram, the two together will furnish an ample explanation of the manner in which the strains are produced and estimated.

TABLE III.

Weight at	Bars.								
	EM.	MD.	DL.	LC.	CK.	KB.	BH.	HA.	AF.
M.. ..	..	−5·8	+5·8	− 5·8	+ 5·8	− 5·8	+ 5·8	− 5·8	+ 5
L.. ..	..	..	..	− 5·8	+ 5·8	− 5·8	+ 5·8	− 5·8	+ 5
K.. ..	..	..	..	..	..	− 5·8	+ 5·8	− 5·8	+ 5
H.. ..	..	..	..	..	..	..	..	− 5·8	+ 5
Total..	..	−5·8	+5·8	−11·6	+11·6	−17·4	+17·4	−23·2	+20

From this Table, and that given in the last chapter, can be perceived, at once, the difference which results in the strains upon the bars, from a different disposition of the load, and, so far as they are concerned, the relative economy of the two principles of loading a girder is apparent. In the first place, the total strain upon all the diagonal bars—in other words, upon the web of the girder—is the same for both conditions of loading. Summing up all the separate total strains upon the diagonal bars, in the Tables given in the last and the present chapter, they are equal to 92·8 tons. At first sight it would therefore appear that, so far as the web is regarded, it is a matter of indifference which principle of loading is adopted. But a little consideration will show that as the web consists of struts and ties, that is, diagonal bars subjected to respective strains of compression and tension, and as wrought iron is stronger under

one of these strains than the other, it is the total amount of the strains upon the struts and ties separately that must be considered, in order to determine to which side economy belongs. With the load upon the upper flange, as in Fig. 37, the sum of the total compressive and tensile strains, upon the diagonal bars of the web, are equal to one another—equal in the present instance to 46·4 tons. But with the load upon the bottom flange, as in Fig. 38, the sum of the compressive strains is equal to 34·8 tons, and the tensile to (92·8 − 34·8) = 58 tons. As iron is weaker under a compressive strain than under a tensile, the less the strain upon the struts in a web, the better for the economy of the structure. With the loading upon the bottom flange, therefore, the aggregate strain upon the struts will be a minimum, and consequently that upon the ties a maximum. The saving effected will therefore depend upon the relative magnitude of these separate strains. In the comparatively trifling example under notice, the actual difference in the amount of metal required, would be too insignificant to take into account, but the general principle holds for larger examples, where the saving in metal would be deserving attention. From the diagram, it can be deduced that mathematically, the strain upon any bar may be found as follows :—Let W = the weight at any apex, N = number of weights situated between the given bar and the centre of the girder, θ = angle of inclination of the bars to the horizon, and S = strain required. Then, accordingly as the bar is a strut or tie, $S = \pm N \times W \times \text{cosec.}\ \theta$. The bar H A has a total strain upon it by geometrical calculation of − 23·2 tons. In this case N = 4, W = 5 tons; cosec. θ = 1·1547, and S = − 23·09, which is practically the same result. As the girder is uniformly loaded,

if L be the distance in feet from the top of the bar to the centre of the girder, and W^1 the load per running foot in tons, or decimals of a ton, the equation may be written, $S = \pm \frac{L \times W^1}{\text{sine } \theta}$.

Passing on to the strains upon the flanges, they are similar to the example when the loading is on the upper flange, two descriptions of strains being induced, one, which may be termed the direct, and the other the transferred action of the weights. If the means of connecting the web and flanges at M be supposed to be a single pin, then M is pushed by the bar E M, and pulled by D M, the action of both being to bring about the same result —namely, the stretching the central portion of the flange M N. It has been demonstrated that the push of E M, that is, its own compressive strain, is equal to zero, so that, in the first place, the portion of the flange M N is stretched by the action upon the pin at M, from the bar D M. This tensile strain is equal to $a\ b = 2 \cdot 9$ tons, in the triangle of forces $M\ a\ b$, the manner of obtaining which has already been shown. Similarly, the remaining triangles, $L\ a\ b$, $K\ a\ b$, $H\ a\ b$, will give $2 \cdot 9$ tons $= a\ b$ = tension upon L M, K L, and H K, and the sum of these will be the total tension upon M N, from the direct action of the load. Consequently this strain $= -(2 \cdot 9 \times 4) = -11 \cdot 6$ tons; that upon L M $= -(2 \cdot 9 \times 3) = -8 \cdot 7$ tons, and so on for the parts K L and H K. Let the strains upon the flanges arising from the action of the bars, or by transference, be now calculated, commencing with the bottom flange. The strain upon each of the bars D M and D L, which are pairs, equals M b = D b = D c. Produce the bar D L, making L b = D c = $5 \cdot 8$ tons; draw $b\ c$ parallel to L C,

meeting the lower flange in c; then L c equal the strain upon L M and M N, resulting from the transference of the weight at M down the bar D L. The weight being ultimately transferred to the abutments by the bars C K and B H, causes strains upon K L and H K, which are all referred to the centres, and consequently pull upon the parts of the flange L M and M N to the same extent. The total strain brought upon M N from the weight at M, during its passage, so to speak, towards the abutment, is equal to the pull at L + the pull at K, + the pull at H. Each of these pulls is equal to a tensile strain of 5·8 tons, and therefore the strain upon M N $= -(3 \times 5 \cdot 8) = -17 \cdot 4$. Again, the strain upon M N from the weight at L, by similar reasoning, $= 2\,\mathrm{L}\,c = 2\,\mathrm{K}\,c = 2\,\mathrm{H}\,c = (2 \times 5 \cdot 8) = -11 \cdot 6$. That from K $= -(1 \times 5 \cdot 8) = -5 \cdot 8$; and that from the weight at H $= 0 \cdot 0$ tons. Summing up, we have the total tensile strain upon

$$\mathrm{M\,N} = \{(4 \times 2 \cdot 9) + (6 \times 5 \cdot 8)\} = -(11 \cdot 6 + 34 \cdot 8) = -46 \cdot 4 \text{ tons.}$$

It will not be necessary to recapitulate the manner, in which the strains upon the other portion of the flanges L M, K L, and H K are obtained, as they are calculated in precisely the same way as in the example in the last chapter. An inspection of the diagram, a measurement of the strains by the scale, and a comparison with the results in Table IV. will be sufficient to explain the whole problem.

To obtain the strains upon the top flange, and commencing with the central portion D E, the first strain brought upon it is due to the weight at M, and caused by the pull of the bar M D, compressing D E towards E. Produce the bar M D; make D b = M b; draw b c

parallel to the top flange to meet the bar D L produced in c; then $b\ c$ = first strain upon D E = 5·8 tons. This strain is transferred to the abutment, and on reference to the diagram it will be readily perceived that the total strain upon D E, resulting from the action of the weight of 5 tons at—

$$M = 3\,bc + ab = 3 \times 5\cdot8 + \frac{5\cdot8}{2} = (17\cdot4 + 2\cdot9) = 20\cdot3 \text{ tons.}$$

Again, the second strain upon it, from the weight at L = (2·5 × 5·8) = 14·5 tons. From the weight at K, we have a strain of (1·5 × 5·8) = 8·7 tons, and from weight at H a strain of (0·5 × 5·8) = 2·9 tons. Summing up, the total compressive strain upon D E = (3·5 × 5·8) + (2·5 × 5·8) + (1·5 × 5·8) × (0·5 × 5·8) = 8 × 5·8 = 46·4 tons, or exactly equal to the strain upon the lower flange. So far as the strains upon the flanges are concerned, it is in this instance of no consequence whether the load be uniformly distributed over the top or the bottom flange.

There are many practical reasons why the lower flange is generally selected as that upon which to place the load; but the choice must depend upon other circumstances than those of simple theory, and cannot be decided upon universal principles. It is sometimes advantageous to have the girders of a bridge placed altogether underneath the level of the rails, so that no vertical projection should break the evenness of the platform; but the more usual plan is to lay the rails upon cross beams, placed upon the bottom flange between the main girders. It has been shown that the total strain upon the bar A F is equal to a vertical compression of 20 tons, equal to the whole load upon the half girder, and that the strain upon the diagonal bar A H was one component of it. The

other component is manifestly the horizontal strain upon A B, and may be always found by multiplying the half load upon the whole girder, or the vertical strain upon A F, by the cotangent of the angle of inclination of the bars to the horizon. Thus the cotangent of $60° = 0 \cdot 57735$ and $0 \cdot 57735 \times 20 = 11 \cdot 54$ tons, or practically the strain upon A B, found by the diagram.

TABLE IV.

Weights at	Parts of the Flanges.								
	AB.	BC.	CD.	DE.	FH.	HK.	KL.	LM.	MN.
M.. ..	+ 2·9	+ 2·9	+ 2·9	+ 2·9	−0·0	− 0·0	− 0·0	− 0·0	− 2·9
	+ 0·0	+ 5·8	+ 5·8	+ 5·8	−0·0	− 5·8	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 0·0	+ 5·8	+ 5·8	−0·0	− 0·0	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 0·0	+ 0·0	+ 5·8	−0·0	− 0·0	− 0·0	− 5·8	− 5·8
L	+ 2·9	+ 2·9	+ 2·9	+ 2·9	−0·0	− 0·0	− 0·0	− 2·9	− 2·9
	+ 0·0	+ 5·8	+ 5·8	+ 5·8	−0·0	− 5·8	− 5·8	− 5·8	− 5·8
	+ 0·0	+ 0·0	+ 5·8	+ 5·8	−0·0	− 0·0	− 5·8	− 5·8	− 5·8
K	+ 2·9	+ 2·9	+ 2·9	+ 2·9	−0·0	− 0·0	− 2·9	− 2·9	− 2·9
	+ 0·0	+ 5·8	+ 5·8	+ 5·8	−0·0	− 5·8	− 5·8	− 5·8	− 5·8
H.. ..	+ 2·9	+ 2·9	+ 2·9	+ 2·9	−0·0	− 2·9	− 2·9	− 2·9	− 2·9
Total ..	+11·6	+29·0	+40·6	+46·4	−0·0	−20·3	−34·8	−43·5	−46·4

In Table IV. are given the results of the action of the different weights, placed at the various apices, upon the separate parts of the flanges, and the Table is so arranged that the strains brought upon each portion of the upper and lower flanges, by each weight, can be distinctly traced without being confused with one another. The strains upon the central portions of the flanges, may be checked by the method of calculation. The central strain is given by the simple formula—

$$S = \frac{W \times L}{8 \times D} = \frac{40 + 80}{8 \times 8 \cdot 66} = \frac{400}{8 \cdot 66} = 46 \cdot 2 \text{ tons},$$

which is an approximation sufficiently near for all practical purposes. With the load situated upon the upper

flange in the example given in the last chapter, the total strain upon the lower flange was equal to $-$ 46·4 tons, or precisely the same as that found in the present instance in Fig. 38. Although the disposition of the load is undoubtedly different, yet the reason of the equality in the aggregate strain is easily to be explained. The actual weight tending to exert strain upon the lower flange, with the weight at the top, was 17·5 tons; but in Fig. 38 the actual weight suspended from the bottom flange is 20 tons. How, then, is the equality to be accounted for? The diagram will point out that the weight at H does not affect the lower flange by any transferred strain, and the only action it induces is the pull, equal to $a\ b$, the vertical component of which is 2·5 tons. The other weights are altogether equal to 15 tons, so that although there are 20 tons upon the flange, yet there are only 17·5 really affecting it, and thus the strains are identical with those produced in the other example. Not only is the central strain the same, but the separate strains upon the parts H K, K L, L M, M N, are identical with those given in the Table of strains in the last chapter.

There are two angles usually selected in practice for the inclination of the bars in the web to the horizon; they are 60° and 45° respectively. The cases already considered have their bars at an angle of 60°. It is worth examining a little into the effect produced by altering their angle to one of 45°, keeping every other item constant relating to the span, depth, and rate of uniform loading. In Fig. 39 is represented the same girder as in Fig. 38, drawn to a scale of 13·33′ to 1″, the depth being, as before, 8·66′. The girder is divided by the central line A B, and in the one half, the bars are inclined at the angle of 60°, and in the other, at that of

45°. In this comparison both the halves of the girder belong to the Warren type, that is, they have only one series or system of triangulation, whereas the lattice, or type possessing two or more series, is that preferred in practice.

On referring to the figure, it will be seen that the angle of 45° divides the half girder into fewer spaces, and consequently there are but about half the number of bars required. On the other hand, the bars are individually rather longer than those inclined at the angle of 60°, but the total length will be less. If the total length of all the bars on both halves of the girder be measured, that of those inclined at the angle of 60° will exceed the length of those at 45° by about two and a half times the length of one of the former. Manifestly the uniformity of loading of the girder is better ensured by the arrangement of bars between A and C, but, as has been already remarked, this object is fully obtained by the introduction of two or more series of triangles. Otherwise, the apices of the triangles between A and D would be too far apart, to allow of the collection of the weights at them, to be regarded as equivalent

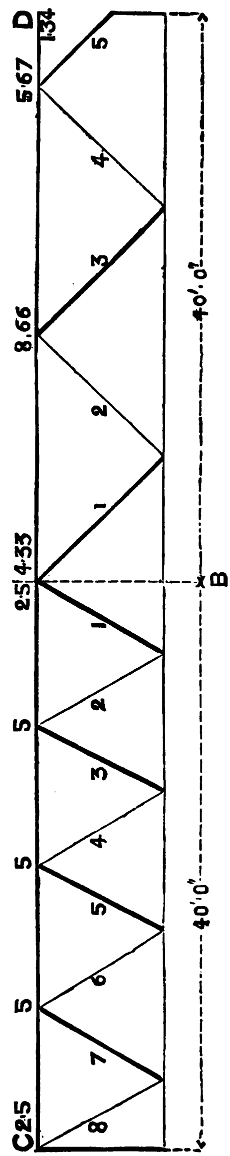


Fig. 39.

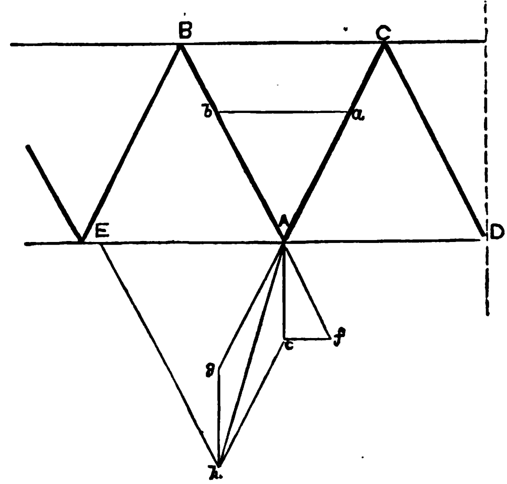
to a uniform distribution of the load. The arrangement of the load upon the apices is shown in Fig. 39. Were the weights situated upon the apices of both halves of the girder identical, the strains upon the bars would vary as the cosect. of the different angles, that is, as $\frac{1 \cdot 414}{1 \cdot 154}$. A strain of 1 ton, upon a bar inclined at an angle of 60°, would become 1·22 tons upon one inclined at 45°. The sum of the strains upon the central part of the flanges will be the same, so long as the span, loading, and depth are constant, as will be apparent from the next chapter, in which the strains upon a half girder with the bars inclined at an angle of 45° will be treated of.

CHAPTER XIII.

THE LATTICE GIRDER. LOAD UNIFORMLY DISTRIBUTED.

In order to render the determination of the strains, upon the various members of a Warren girder, perfectly clear to our readers, each step of the graphical process was successively explained in the former chapters. It will now be seen that it is not absolutely necessary to find every strain individually, provided the sum of those of the same kind—that is plus or minus—be accurately obtained. Let the diagram in Fig. 40 represent a portion of a Warren or primitive type of the lattice girder, with one series of triangles, loaded upon the bottom flange. The general case, applicable by simple repetition and summation to the whole of the flanges and web, consists of a weight at A, and a strain transferred along the bar A C to the point A. The magnitude of this strain will vary with the number of weights situated between the point A, and the centre of the girder, and will never exceed

Fig. 40.



the sum of these weights, multiplied by the cosecant of the angle of inclination of the bars of the web. Of these *c f* and A *f* are due to the action of the weight at A, and *a b* and A *b* to the strain transferred along the bar A C. The strains to be found, are those upon A D and A B. It has been already shown, that the total strain upon A D, equals $c f + a b$, and that upon A B, equals A *b* and A *f*. To obtain the sum of $c f + a b$ and $A b + A f$ we proceed as follows:—Let the weight suspended from A equal 1 ton, and the strain transferred to the same point, in the direction of the bar A C, equal $1\frac{1}{2}$ tons. Let A *c* and A *g* represent these strains both in amount and direction. It is manifest that they may be regarded as two distinct forces, acting in different directions upon the point A, and may consequently be replaced by a single force, or resultant. To find this, construct the parallelogram of forces by drawing *c h* and *h g*, parallel to the two original forces, and join the points A and *h*, by the line A *h*. The line A *h* represents, in magnitude and direction, the resultant of the two forces or strains acting upon the point A. If from the point *h*, we now draw *h* E parallel to the bar A B, the line *h* E will equal the total strain upon the bar A B, and A E that upon the part of the flange A D. If these lines be checked *h* E will be found equal to $A f + A b$ and A E equal to $c f + a b$.

The type of girder hitherto selected for illustration is not so well adapted for actual practice as the lattice, which embraces in the web several series of triangles. At home, a Warren girder is never put up now, although from the facility with which it can be erected, it is frequently employed abrŏad, and in situations where it is difficult to procure skilled labour. From the small

amount of lateral rigidity, possessed by the unintersected bars of the web it is not suited for any but very moderate spaces. We may therefore pass from the consideration of the Warren to that of the lattice type, in which one or more intersections occur in the length of the diagonal bars. In Fig. 41 is represented the skeleton elevation of a lattice girder, with three series of triangles A, B, and C, which, practically, are totally independent of one another. This statement requires some explanation. The load is supposed to be uniformly distributed over the top of the whole girder, and consequently, equal portions of it are situated upon the several apices of each system of triangulation, or series of triangles. Let us take the case of the weight placed upon the apex A^1. It is conveyed upon the principle of the lever, to each of the abutments D, by means of the bars A^1 A, A A, A D, A^1 A^1, A^1 A^2, A^2 A^2, A^2 A^3, A^3 D, and is considered to produce no practical effect upon the bars of the other systems B and C, at the points of their intersections with its own system A. It is assumed that whether pins or rivets be employed, for connecting the diagonal bars of the different systems, they do not act as a medium for the transference of strain, but simply bind the bars together, or hold them in a vertical plane. Theoretically, there is no doubt but that some slight strain is induced, but as we are dealing with the subject principally in a practical light, we shall not discuss the question at present.

There are two methods of arriving at the strains upon the web of a lattice girder, an approximate and an exact one. The former consists, in supposing the total load distributed upon the apices of one system only, determining the strains, as in a Warren girder, and dividing them by

the number of series or systems of triangles. This method is, in fact, equivalent to regarding the lattice as a simple Warren girder, and determining the average strain for each bar, from the maximum strain resulting upon one. Where the girder is small, and the number of systems of triangles does not exceed a couple, and the bars are pretty close together, this approximate method may suffice, but it should never be used in examples upon a large scale, or where it is desirable to obtain a very accurate result. Referring to Fig. 41, and supposing the bars A A, A A^1 to be pairs, the strain upon either, divided into three parts, would not afford an accurate result for each of the corresponding bars of the other systems. To ascertain the strains not only upon the bars of the web of a lattice girder, but also upon the different parts of the flanges, with that proper amount of care and accuracy which should ever accompany all such calculations, there is but one true method, and that is the one which has been already explained in the analysis of the Warren girder in the previous chapters. In Fig. 41 the load should be considered as uniformly distributed over all the apices, and the strains resulting from each separate weight obtained, and tabulated in the manner already

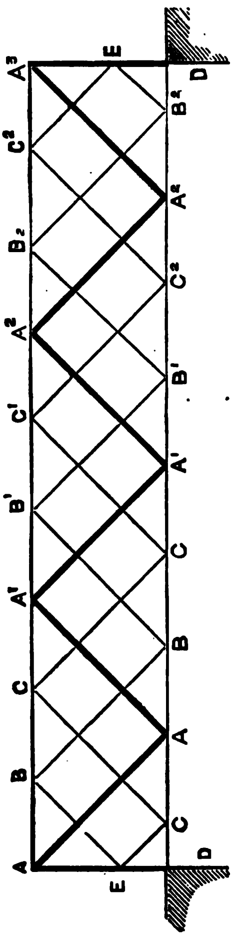


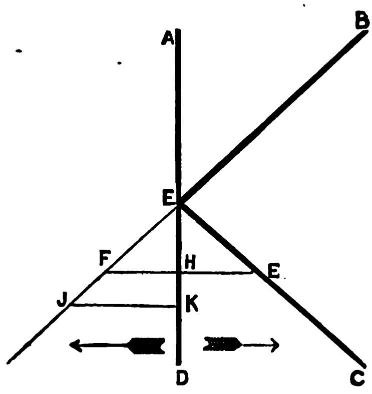
FIG 41.

described. It would be an unnecessary repetition to give the analysis of the strains upon each bar and portion of the flanges of the example in Fig. 41, as by taking the action of each weight separately the problem becomes reduced to the case of a Warren girder, the only difference consisting in the greater number of bars, and, consequently, the greater number of individual strains to be tabulated. In applying the mathematical formula already given, to the determination of the strain upon any bar of a lattice girder, care must be taken to include only so much of the load situated between the bar and the centre of the girder, as is placed upon the apices of that particular system of triangles to which the bar belongs. So far, there appears to be no difference in the general disposition, and nature, of the strains induced upon both the Warren and the lattice girder. The latter is nothing more than a more elaborate, or, as perhaps some of our younger readers might term it, more troublesome example of its theoretical prototype.

If we examine carefully into the strains brought upon the vertical ends of the girders, or pillars, as they are usually called, a notable difference will be perceived. Confining our attention to the one system of triangles in the elevation in Fig. 41, which commences at A over one abutment, and terminates at A^3 over the other, we may consider it to represent a Warren girder. Under these circumstances, it has been already demonstrated that the only strain brought upon the end pillars A D, A^3 D, is a vertical one, which is equal, for each pillar, to half the total load upon the girder. In fact, the pillar is in precisely the same condition as if it directly supported half the load. This is worth remarking, as it points out that any vertical load will be ultimately transferred to the

points of reaction, without altering its original value at those points, notwithstanding the manner in which it may be transferred, and the number of strains to which it may give rise in the various bars, which may be considered to act as the medium of its transference. The total load, being uniformly distributed, is conveyed in equal portions to each abutment, and the vertical reaction which causes the strain in the end pillars is equal to exactly half that load. Again, referring to Fig. 41, and regarding it in its true light, as the elevation of a lattice girder with three systems of triangles, it will be seen that two bars B E, C E, are connected to the pillar A D, and two others C^2 E, B^2 E, to the pillar A^3 D. Consequently, the strains upon these bars must be resisted by the pillar, and it remains to ascertain of what nature are the strains which the pillar undergoes. It will be quite sufficient to examine into the case of one pillar—that over the abutment on the left, or A D. Of the two bars, one, B E, is a strut, and the other, C E, a tie; consequently their strains may be resolved into their components—one in a vertical, and the other in a horizontal direction.

Fig. 42.



This is shown in the diagram in Fig. 42, in which A D is the vertical pillar, and B E, C E the strut and tie. Let the compressive strain upon BE be equal to 1 ton; and since this tends to push the pillar A D outwards, produce the bar B E beyond the pillar, and lay off upon it the

distance E F, equal to 1 ton, draw F H to meet the pillar at H; then F H equals the horizontal component of the compressive strain upon the bar, and tends to push the pillar outwards. It is, in fact, the transverse strain to which the pillar is subjected by the oblique thrust of the bar B E. Now suppose the bar C E to be under a tensile strain of 1 ton, it will evidently pull the pillar inwards, with the same amount of force with which B E pushes it outwards. Making E F equal to 1 ton, and drawing F H to meet the pillar, it is plain that the horizontal components—being equal and in opposite directions—balance one another, and there is in that case no transverse strain brought upon the pillar. But if the strain upon one bar exceed that upon the other, then the transverse strain is equal in amount to the difference of their horizontal components. For instance, let the strain on B E be 1·5 tons; make it equal to E J in the diagram, and J K will represent the horizontal component, tending to thrust the pillar outwards, while the pull in the contrary direction is represented by E H. The difference between their respective values will evidently give the measure of the transverse strain upon the pillar. These horizontal components, which in the case of the intermediate bars B E, C E, must be resisted by the pillar A D, are analogous to those which at the top and bottom of the pillar, cause compressive and tensile strains respectively upon the upper and lower flanges.

The object of departing from the simple Warren girder, and introducing secondary systems of triangles is threefold. First, in the lattice type, the points of attachment between the upper and lower flanges are multiplied, and a more uniform distribution of load

thereby ensured; secondly, the flanges are rendered incapable of undergoing any strains, save those of longitudinal tension and compression; and thirdly, the web itself is made a great deal stiffer, and better adapted for the case of deep girders. It is evident that if the apices be too far apart, that is, if there be not a sufficient number of series of triangles, the assumption that a uniformly distributed load may be considered collected upon the apices, will not hold good, and the assumption becomes still farther from the truth, in the case of a moving or variable load. Each portion of the flange, between the apices or points of attachment of the web, becomes in reality a short girder, and the entire flange approaches the conditions of a continuous girder. At the same time, since the real economy of the lattice form is to be found in its web, the bars must not be placed too near each other. In other words, there must not be too great a number of separate triangulations. As a rule, this is the great mistake made in designing lattice girders. The bars are frequently placed so near together, that the true principle of a lattice web is departed from. The particular advantages resulting from its adoption are lost, and it would be just as cheap to construct a plate girder instead.

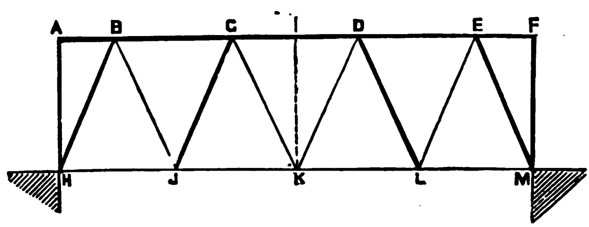
CHAPTER XIV.

THE LATTICE GIRDER. MOVING LOAD.

Hitherto, the strains investigated have been those resulting solely from a uniformly distributed load; but, with very few exceptions, girders are subjected to the action of a variable or moving load—the weight of a train, for instance. In some cases the moving load will very much exceed the fixed or permanent weight of the bridge itself, while in others it will bear but a small proportion to it. There are two points to be considered with respect to a moving load. The one is the actual strain resulting from it, and the other, the effect it produces by its impactive or concussive force, which cannot be accurately, or even approximately ascertained. Nevertheless, some allowance must be made for it, especially in bridges of small span, where the *vis inertiæ* of the superstructure presents but a feeble resistance to the shock of an express train. In the early days of railways, it was not an uncommon practice, actually to load the bridges with an additional depth of ballast, with the view of increasing its insistent weight, and so diminishing the vibration caused by the transit of a heavy train. However well this plan may speak for the precautionary principles entertained by the engineers of those days, it does not say much for their ideas of economy of material, or their skill in designing iron structures. Paradoxical as it may appear, the placing of a greater load upon a portion only of a girder, reduces instead of increasing

the strain already existing upon some of the bars of the web, increases those upon others, and sometimes changes their character from compressive to tensile. Let Fig. 43 represent a Warren girder in elevation uni-

Fig. 43.



formly loaded upon the upper flange. The thick lines show the struts or bars in compression, and the thin those in tension. Suppose we now illustrate the action of a moving load by placing an additional weight on the apex D, and examine into the result. The portion of it conveyed to the abutment F M, will compress the bars D L, E M, and F M, and stretch L K, L E, and M L. But these are the normal conditions of these bars under the existing permanent load, therefore their strains will be increased. The contrary takes place on the other side of the centre line of the girder. Under a uniform load, the bars D K, C K are practically free from strain, but the portion of the moving load situated at D, which is transferred to the abutment A H, compresses the bar D K, and stretches C K. It is readily perceived that when the variable load, or a part of it, is situated at C, then the bar C K will be in compression, and D K in tension. Both these bars are therefore liable to strains of alternate tension and compression, according to the position of the movable load.

The simplest practical method of calculating the

strains upon a girder, due to a passing load of uniform weight, will be first of all to calculate the strains upon the assumption that the total load upon the bridge, including its own weight, is uniformly distributed over the span. One advantage of this method is, that we at once obtain the maximum strains upon the different parts of the upper and lower flanges, since they take place when the passing load covers the whole span, that is, in reality, when it is uniformly distributed. The strains having been calculated upon this assumption, the design of the girder can be proceeded with; and in the meantime, the effect of the moving load upon the various bars, obtained by a graphical diagram, can be allowed for, by increasing their dimensions, if necessary, or counterbracing them, as may be required. By counterbracing any portion of a structure, is meant, bracing it in such a manner, as will enable it to resist a strain of compression as well as one of tension. That it is absolutely necessary to regard the moving load also as a uniformly distributed one, is evident from the fact that a train may at any moment be brought to a standstill upon a bridge, and thus cover the whole span. The other method will, of course, give the same results. This would consist in, first of all, deducing the strains resulting from the absolute permanent weight of the bridge itself, and afterwards finding those due to the passing load. In reality, however, if the first method be employed, it will be at once seen that the load subsequently considered as a moving one, causes very little alteration in the proportions of the bars, and none whatever in some of them. To elucidate this thoroughly, we must recapitulate a little, and will take the example given in Chapter XII. on this subject, where tables of the strains upon the various parts are

inserted. As already stated, the strains upon the flanges will not alter, and it is therefore only the diagonal bars we have to regard. In the example referred to, the girder is 80′ in span, and supposed to be loaded uniformly upon the bottom flange with 40 tons, or at the rate of half a ton per foot run. This load of 40 tons will now be considered as moving, that is, approaching the girder at one extremity and gradually advancing over its entire length. Upon the assumption that the load was uniformly distributed, or what amounts to the same, that it covered the whole span, the strains upon the diagonal bars of the web were given in the following Table:—

TABLE V.

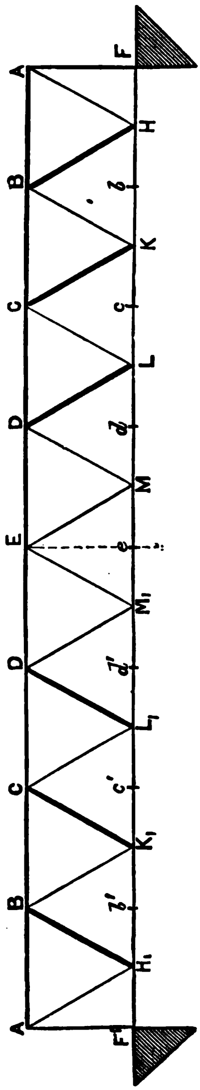
Weight at	Bars.								
	EM.	MD.	DL.	LC.	CK.	KB.	BH.	HA.	AF.
M.. ..	..	−5·8	+5·8	− 5·8	+ 5·8	− 5·8	+ 5·8	− 5·8	+ 5
L.. ..	..	..	..	− 5·8	+ 5·8	− 5·8	+ 5·8	− 5·8	+ 5
K.. ..	..	..	..	..	..	− 5·8	+ 5·8	− 5·8	+ 5
H.. ..	..	..	..	..	..	..	..	− 5·8	+ 5
Total ..	..	−5·8	+5·8	−11·6	+11·6	−17·4	+17·4	−23·2	+20

It now remains to determine whether the proportions given to the bars, are sufficient to enable them to withstand the effect of the same load, situated successively at different points of the girder.

Let a full skeleton elevation of the half girder in Chapter XII. be represented in Fig. 44, and let the moving load be supposed to be advancing from the right. The method of arriving at the various strains is precisely similar to that already adopted, and amply explained in the diagrams, and graphical analysis contained in previous chapters. It will, therefore, be unnecessary to repeat the more elementary steps, but merely give the

rationale of the process, and leave our readers to verify the accuracy of the results for themselves. It is necessary to work out the strains upon the full elevation in this instance. The half girder will not suffice, as the strains resulting from these portions of the load, situated upon each side of the centre of the girder, only counterbalance each other, when they cover the whole span, that is, when the weights upon each half are identical. It is true that in a girder of large span, if the centre of gravity of the whole train be exactly over the centre of the girder, although the whole length of it might not equal the span of the bridge, the same compensation of strain would occur, but this is more a theoretical than practical question. And again, if the division were so adjusted that the weights were equal, the distances occupied by those weights upon each side of the centre would not be equal. In Fig. 44 let the moving load be advancing from the right-hand side, and suppose it to cover so much of the girder as extends to the point b, which will be equivalent to supposing a weight of 5 tons, supported at the lower apex of the bars A H and H B.

Fig. 44.



Let us now examine how this load will affect the various bars throughout the web. As there is no load situated upon the other portion of the girder,

there are no counterbalancing strains so far as the moving load is concerned, and consequently the strains must be deduced from the principle of the lever. Of the 5 tons situated at H, one-sixteenth is transferred to the far abutment, and fifteen-sixteenths to the near. Therefore $\frac{15 \times 5}{16} = 4 \cdot 6875$ tons are transferred to the near support, and $0 \cdot 3125$ tons to the most distant. If these weights be laid down to scale in the manner already shown, the strains resulting from them upon all the bars, situated between the point H, and the supports to which they are respectively transferred, can be readily ascertained. Again, when the moving load reaches the point c, and the weight is supposed to be at K, the proportions into which it is divided and conveyed to the supports are $\frac{13 \times 5}{16} = 4 \cdot 062$ and $\frac{3 \times 5}{16} = 0 \cdot 938$ tons respectively, and so on, for all the other points d, e, d^1, e^1, b^1, until the load covers the whole girder. If W represent that portion of the load transferred to either abutment, then the strain either of compression or tension, resulting in consequence upon all the bars, situated between the point where the load is assumed to be placed, is equal to W $\times$ cosecant of angle of inclination of the bars, equal in this instance to $W \times$ cosecant $60° = W \times 1 \cdot 154$. The maximum strain upon the vertical pillar A F, or the bar A H, occurs when the load covers the whole span. The only manner in which these bars can be affected is from the pull upon the point H, which consists of the action of the successive weights as they are finally conveyed to the supports. Consequently, the simple summation of a series will give the maximum strains upon these bars. Thus from the weight at H of 5 tons,

we have, $5 \times \frac{15}{16}$ transferred to the near abutment; from that at K, $5 \times \frac{13}{16}$, and in like manner for the rest. Summing up we have

$$\frac{5}{16}(15+13+11+9+7+5+3+1) = \frac{5}{16} \times 64 = 20 \text{ tons.}$$

Referring to Table V., it will be seen that this is precisely the strain found before for the vertical pillars. Similarly for the end diagonals A H. As they can only be affected by the vertical component of the strains at H, their maximum strain is given by $20 \times 1 \cdot 1547 = 23 \cdot 1$, or practically the same result obtained before. In Table VI. are shown the strains upon the various bars, resulting

TABLE VI.

Position of load.	Bars.							
	A H.	H B.	B K.	K C.	C L.	L D.	D M.	M E.
H	−5·41	−0·36	+0·36	−0·36	+0·36	−0·36	+0·36	−0·36
K	−4·68	+4·68	−4·68	−1·07	+1·07	−1·07	+1·07	−1·07
L	−3·96	+3·96	−3·96	+3·96	−3·96	−1·80	+1·80	−1·80
M	−3·24	+3·24	−3·24	+3·24	−3·24	+3·24	−3·24	−2·52
M_1	−2·52	+2·52	−2·52	+2·52	−2·52	+2·52	−2·53	+2·52
L_1	−1·80	+1·80	−1·80	+1·80	−1·80	+1·80	−1·80	+1·80
K_1	−1·07	+1·07	−1·07	+1·07	−1·07	+1·07	−1·07	+1·07
H_1	−0·36	+0·36	−0·36	+0·36	−0·36	+0·36	−0·36	+0·36
Total ..	−23·04	+17·27	−17·27	+11·52	−11·52	+5·76	−5·76	+0·00

Position of load.	Bars.							
	EM_1.	M_1D.	DL_1.	L_1C.	CK_1.	K_1B.	BH_1.	H_1A.
H	+0·36	−0·36	+0·36	−0·36	+0·36	−0·36	+0·36	−0·36
K	+1·07	−1·07	+1·07	−1·07	+1·07	−1·07	+1·07	−1·07
L	+1·80	−1·80	+1·80	−1·80	+1·80	−1·80	+1·80	−1·80
M	+2·52	−2·52	+2·52	−2·52	+2·52	−2·52	+2·52	−2·52
M_1	−2·52	−3·24	+3·24	−3·24	+3·24	−3·24	+3·24	−3·24
L_1	−1·80	+1·80	−1·80	−3·96	+3·96	−3·96	+3·96	−3·96
K_1	−1·07	+1·07	−1·07	+1·07	−1·07	−4·68	+4·68	−4·68
H_1	−0·36	+0·36	−0·36	+0·36	−0·36	+0·36	−0·36	−5·41
Total ..	0·00	−5·76	+5·76	−11·52	+11·52	−17·27	+17·27	−23·04

from a moving load, which is successively situated upon the points H, K, L, etc., and is equal to 5 tons at each point. In other words, it successively covers the portions F b, F c, F d, F e, F d^1, F c^1, F b^1, F F^1, of the girder in Fig. 44 at the rate of half a ton per foot run.

Adding up the totals of the strains, that is, taking the algebraical sum of the different signs, we obtain the result on the supposition that the load is covering the whole girder. If we compare these totals with those in Table III. they will be found to approximate near enough for practical purposes, and thus to demonstrate the truth of the analysis. The greatest difference does not exceed 0·16 of a ton. The object of the calculation of Table VI. is not, however, merely to prove the accuracy of the investigation, but to indicate the manner in which the various bars are affected. In Table III. it will be seen that the strains upon the bars are all of the same character, but in Table IV. they are subjected, according to the position of the load, to both descriptions of strain. If we take the bar H B, or B H^1, which is really a strut, we find it has a small tensile strain, when the load is at A, of 0·36 tons, and B K, which is a tie, is subjected to a compressive strain of the same nature. Consequently, when the moving load is considerable, and these different strains become of large amount, the bar must be counterbraced, that is, made of such a section as will resist the one strain as well as the other. It must be borne in mind that with a moving load, it is not any one particular position of it which is only to be provided for, but every position in which it can affect the strains upon the diagonals. Looking at the central bars, M E, E M^1, which under the uniform load, have no strain upon them, we perceive that they are subjected to a compres-

sive strain at one time and a tensile at another, accordingly as the load advances over the girder, and must therefore be of such a section and strength as will resist these strains. The necessity of working out a table of strains, in order to arrive at a correct estimate of the effect of a moving load upon the bars of the web, is now apparent. It is commonly assumed that there is no strain upon the central bars of the web, simply because that is the condition obtaining under a uniform load; but Table VI. demonstrates the fallacy of all such loose conclusions.

The present example only includes the case where the load is situated upon the bottom flange; but, from what has been stated, there will not be the slightest difficulty in applying the same principle to the other instance, where the load is placed on the top. There is this difference to be remarked in the two examples:—When the load is at the top, the strains upon both diagonals nearest to the load will be compressive, and tensile when it is upon the lower member. To render the explanation complete, it is necessary to construct another Table, showing the maximum strains of both kinds that the bars are subjected to. This is readily accomplished by adding together all the strains that have the same sign, and tabulating them under their respective bars.

A reference to Table VII. will indicate, at a glance, the relative maximum strains that the bars in Fig. 44 are subjected to, by the action of a passing load of 0·5 ton per foot run. The stroke appended to the bottom of the letters in Fig. 44 is merely to avoid confusion, and will not therefore prevent their being recognised as the same bars as in Table III. If the load be supposed to advance from the opposite end of the girder, of course

the bars will change places so far as the strains are concerned.

TABLE VII.

Bars.	Maximum compressive strain in tons.	Maximum tensile strain in tons.
AH	0·00	23·04
HB	17·63	0·36
BK	0·36	17·63
KC	12·95	1·43
CL	1·43	12·95
LD	8·99	3·23
DM	3·23	8·99
ME	5·75	5·75
EM_1	5·75	5·75
M_1D	3·23	8·99
DL_1	8·99	3·23
L_1C	1·43	12·95
CK_1	12·95	1·43
K_1B	0·36	17·63
BH_1	17·63	0·36
AH_1	0·00	23·04

Table VII. shows that the maximum strain upon any bar takes place when the load covers the longer segment. The maximum compressive strain upon any bar that is a tie, takes place when the load covers the shorter segment, and the maximum tensile strain upon any strut under the same conditions. From this rule must be excepted the two centralbars, which will be affected, accordingly as the load is on the upper or lower flange, in a different manner. The strains upon a lattice girder, with two or more systems of triangulation, resulting from a moving load, can be calculated by the rules already given, bearing in mind, that it is only the diagonals of that particular system, upon the apices of which the load rests, that are affected by the load. The other bars suffer no strain until some of the apices belonging to their own system are loaded. In the present instance the moving load has been considered to be of greater amount than it really is, and the conclusion to be drawn from the

investigation manifestly is, that in bridges of small span, if the strains be calculated upon the assumption that the total load, live and dead, is uniformly distributed over the whole span, there is not much difference occasioned in the strains with the exception of the middle bars, But in practice these bars are generally of the same scantling as those in their immediate vicinity, and are therefore strong enough. The principle to be kept in view is, that if the permanent or dead load bear a very large proportion to the moving or live load, the effect of the latter in augmenting the maximum strains upon the bars will be very trifling. If, on the contrary, it be small, then the moving load will consider ably modify the existing strains. It is a simple question of the preponderance of one load over the other. In practically designing the girder, care must be taken not to cut down the material too fine, especially when providing for the action of a moving load. This is the more necessary, as the strains that are calculated, are supposed to be simply those resulting from a load, successively superimposed upon different parts of the girder. No allowance is made in the theoretical calculation for the violent shock, concussion, and consequent vibration that attend the passage of a heavy train over a bridge. This must be allowed for by experience, by the introduction of such additional bracing as the skill of the engineer suggests. These are points which cannot be learned from books, but which must be the result of actual practical knowledge. It is for this reason that the calculation of strains, and the determination of the sectional area required, should proceed, *pari passu*, with the design and the actual drawing of the girder. It is not sufficient to design a structure that shall merely

resist the forces to which it is subjected. The problem is to design it so that it shall resist them in the best and most economical manner, which can only be ensured by a practical knowledge of what may be termed "the putting together" of ironwork. It would be to little purpose, to give the web of a plate girder the number of square inches required to resist the shearing strain, unless it were stiffened in a manner that would allow of its being able to receive the strain properly. Theoretically speaking, the web might be strong enough, but practically it might be so weak that it would buckle up under a fourth of the calculated strain.

CHAPTER XV.

TRUSSED GIRDER FOR SMALL SPANS.

HAVING fully investigated the subject of the Warren girder and its multi-triangular type, the lattice, and given rules for determining the strains upon any part of them, a peculiar class of girder may be now considered, which is sometimes used, but which is inferior to the examples already adduced. For small spans and for timber bridges it constitutes a handy and convenient form, and is represented in Fig. 45. By placing the loading on the lower flange, the truss itself acts as a hand-rail, and the whole structure has a very compact and neat appearance. In the elevation in the figure, the thick lines represent the parts in compression, and the thin those in tension, and the annexed table gives the strains upon the different bars for one-half of the truss. The truss has a clear span of 40′, a depth of 5′, and the total load which is placed upon the bottom flange is equal to 21 tons. The distribution of this load will be as follows:—At the points D and D^1 there will be 7 tons, and 3·5 tons will be supposed to rest directly upon the abutments. The weight of 7 tons at D will manifestly, in the first instance, be borne altogether by the vertical tie D B, and transferred to the point B, where, upon the principle of the lever, a portion of it will be conveyed to the abutment at A, and another portion to that at A^1. The respective portions will be $\frac{14}{3}$ and $\frac{7}{3}$. The first of these will produce com-

pressive strains in the upper flange B C, and in the diagonal B A, and tensile strains in the vertical B D, and in the parts of the lower flange A D and D C. Similarly, the latter portion of the weight will compress the parts B D^1, B^1 C, and B^1 A^1, and stretch C D^1, D^1 B^1, and A^1 D^1. An analogous action arises from the weight of 7 tons situated at D^1, and the sum of the two gives the total strain upon the various members of the truss. These strains are tabulated in Table VIII.

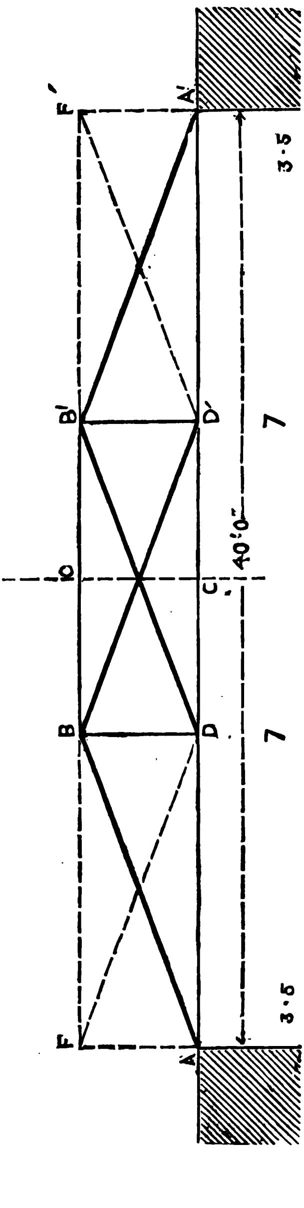
TABLE VIII.

Weight at	Bars.						
	AB.	AD.	BC.	BD¹.	BD.	DC.	DB¹.
D	+13·40	−12·60	+12·60	+6·70	−7	−12·60	..
D¹	+ 6·70	− 6·30	+ 6·30	..	−2·3	−12·60	+6·70
	+20·10	−18·90	+18·90	+6·70	−9·3	−25·20	+6·70

The strain upon the bar A B may be checked by calculation. It is known to be equal to $\frac{W}{2 \sin. \theta}$. To find θ, or the angle B A D, we have tang. $\theta = \frac{BD}{AD}$, or log. tang. θ = log. 5 − log. 13·333 + 10. Solving, log. tang $\theta = 9 \cdot 574021$, and $\theta = 20° \ 33''$. Consequently the strain upon A B $= \frac{7}{\sin. \theta} = \frac{7}{0 \cdot 35102} = 19 \cdot 94$ tons, which is a sufficient approximation to the result obtained by the geometrical process. The dotted lines represent the hand-rail, which has nothing to do with the truss, theoretically considered. If B D, or B^1 D^1, were made struts instead of ties, and the load placed upon the top, then the diagonals, B D^1 and B^1 D, would be in tension instead of compression. Moreover, if the dotted lines

D F and D^1 F^1 were ties under the same circumstances, so that the form of the truss was represented by F, F^1, D^1, D, then with a load only at B, there would be no strain upon the diagonal B D^1. But if there be a load of 7 tons upon B and B^1, and the strains be worked out for the truss represented by F, F^1, D^1, D, they will be found to be equal in amount to those already obtained for the truss A B B^1 A^1, although of an opposite character for the corresponding bars. The strains upon the flanges will be the same, both in amount and character, and they will be a maximum in all cases when the truss is uniformly loaded. The maximum strain that the truss will undergo, will depend upon the position of the load, both with respect to the flanges and the distance from the abutments. In the present instance, if the load were placed upon the top member—that is, upon B and B^1, and both B, D and B^1, D^1 were ties, then either would undergo its maximum strain, when it was itself free from load, and the other

FIG. 45.



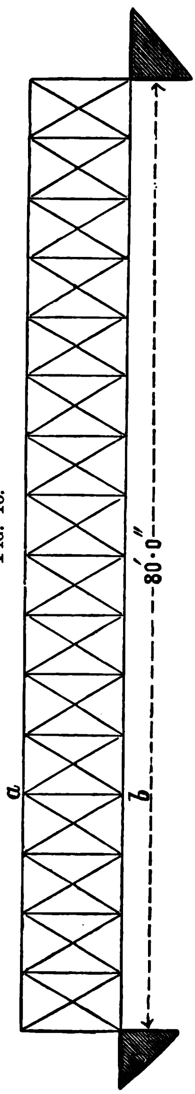
apex loaded. If the load were placed on the lower member under similar circumstances, then the maximum strain would take place when both apices were loaded. Supposing B, D and B^1, D^1 to be struts, the conditions of the maximum strains are exactly reversed under the same methods of loading. It would not make any material difference in a span so small as 40′, so far as the strains are concerned, which form of truss was employed, but there might be a slight gain by making the bars B D and B^1 D^1 struts instead of ties. In that case, the truss would take the shape of F F^1, D^1 D, and all the diagonals would be ties. The truss in Fig. 45 is suitable for either a uniformly distributed or a moving load, as the diagonals intersecting each other at the centre of the truss are counterbraced, and can act either as struts or ties as the position of the load may demand. The strains in Table VIII. are those upon one-half of the truss, resulting from the combined action of the two weights situated at the points D and D^1. The strains upon the other half are precisely similar, but the action of the weights will be reversed. The weight at D produces the greater strains upon the half of the truss, as shown in Table VIII., and that at D^1 the lesser. These conditions are reversed upon the other half of the truss.

The strains upon the other bars in the truss in Fig. 45 may be easily obtained by calculation, when once the angle of inclination of the diagonals with the horizon has been calculated. Thus the strain upon B C or A D is equal to the strain upon B D × cotangent of 20° 33′, equal to $7 \times 2 \cdot 676 = 18 \cdot 73$, or practically the same result as given in Table VIII. The strain upon each of the central braces B D^1, B^1 D, will be equal to the vertical component of their load, multiplied by the cosecant of

their angle of inclination to the horizontal. Their vertical load is $\frac{7}{3}$, or $2 \cdot 66$ tons, and the angle is the same as for the strut A B. The strain equals $\frac{7}{3} \times$ cosecant $20^\circ 33'$, equals $2 \cdot 66 \times 2 \cdot 85 = 6 \cdot 64$ tons. The additional strain upon D C, the centre part of the tie, is due to the strain upon the central brace, which is repeated twice upon D C, between the centre and the abutment.

There is another description of girder which, although inferior to the regular lattice, is nevertheless used a good deal, especially in timber work. The general type is represented in Fig. 46, but there are one or two varieties. In some there are but one set of diagonals and in others two, and they vary in their duty as struts and ties. A distinction must be made between this type of girder, in which any upright, *a b*, really acts as a strut or tie, and uprights similar to those in the railway bridge over the Thames at Charing Cross, which are not of the slightest use whatever, and may be regarded as so much waste of material. It is optional, in the example represented in Fig. 46, to make the verticals struts, and the diagonals ties, or to reverse that arrangement. On the argument solely, that the shorter members should be those subjected to a strain of compression, it would appear that the verticals should be made to act as struts. There are,

Fig. 46.



however, other reasons which render this rule of little value except as a guide. Besides, in the case in question, the diagonals, in consequence of their intersecting half-way, are really shorter than the verticals; for although no strain is supposed to be developed at the intersections, yet it is obvious that a bar could not deflect laterally at that point. The investigation of this form of girder will complete the examples of the straight girder principle.

CHAPTER XVI.

GIRDER WITH VERTICAL STRUTS.

Another type of girder deserving investigation is that represented in Fig. 47, in which the verticals are struts and the diagonal bars ties. It will be seen hereafter, that this system of bracing is only adapted for a fixed load situated at the centre, and that to render it fitted for the purpose of carrying a movable load, it is necessary to introduce certain modifications, which demonstrate at once that it is preferable to use the lattice form instead. As, however, the girder shown in half elevation in Fig. 47 is frequently employed, a general analysis of the strains to which its various parts are subjected, is necessary to the subject. The span of the girder is 80′, and the depth and rate of loading similar to those, adopted in the examples of the Warren and lattice girders already investigated. By maintaining these data constant, the difference between dissimilar types of braced girders will be the more apparent, and, moreover, as with these data, some of the strains are also constant, the one calculation and Tables of strains act in some measure as a check upon the other. In addition to these reasons, the student and intended engineer cannot familiarise himself too much with different forms of iron construction. It not unfrequently happens in the course of his practice, that the circumstances of the case demand the adoption of very peculiar structural shapes, and unless he is thoroughly well acquainted with the manner in which strains act, he will have great difficulty in designing

the form scientifically and economically. In Fig. 47, the girder is loaded upon the top flange at the rate of half a ton per foot run. Consequently there will be a distribution of the load at the apices of the triangles as follows:—At each apex from B to H, there will be a weight of 2·5 tons, at A a weight of 1·25 tons, and at K a weight of 1·25 tons. The total weight at K is 2·5 tons, but as it is only necessary to consider half the girder, we are only concerned with half the weight placed on the central strut. Frequently, in deducing the strains belonging to an example like the one in question, it is sufficient to assume the weight distributed equally over the apices, and thus take 2·5 tons as the load upon each apex from B to K. But as the instances where this latitude may be indulged in, and where it ought not, must be left to the judgment of the engineer, it is better in all cases to adopt the strictly accurate

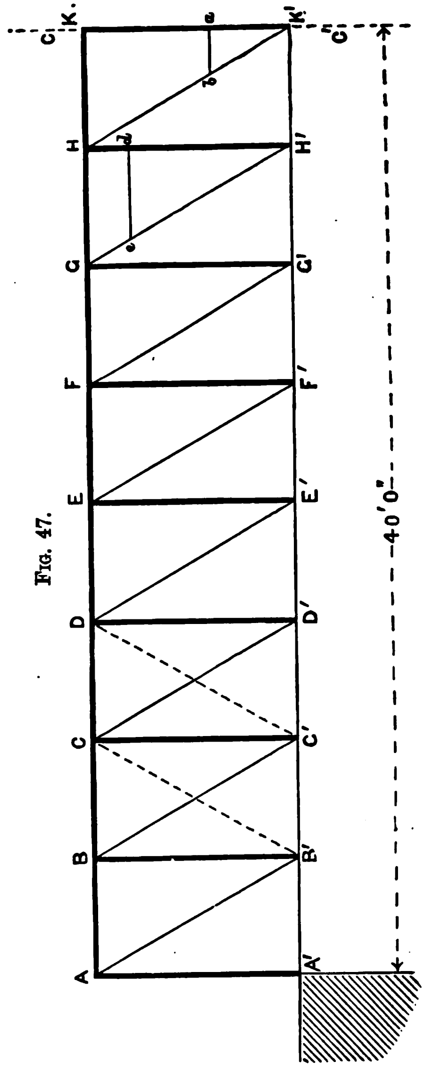


Fig. 47.

plan. Another reason for adhering to accuracy in graphical analysis is, that as small errors will creep in, and gradually accumulate in spite of every care, the results will not check with those of calculation, if the correct data be not rigidly followed throughout.

An examination of Fig. 47 will readily demonstrate that the strains upon the different parts are very easily arrived at. The weight upon the strut K is 1·25 tons. Make this equal to $K^1 a$, and draw $a\ b$ parallel to the bottom flange. $K^1 b$ is the strain upon the diagonal $H\ K^1$, and $a\ b$ that upon the bottom flange due to the weight upon the apex K. These strains are transferred throughout the whole of the bracing to the abutment A^1, causing strains upon both the web and flanges in their passage. Since the weight at H is double that at K, the strains will be also double. Make $H^1 d = 2 \cdot 5$ tons, draw $d\ e$ parallel to the top flange, and the respective strains can be measured off as before. In like manner all the remaining weights may be disposed of, and the sum of the strains at any particular point will give the total strain there. The strain upon any one of the vertical bars, is identical with the shearing strain at the corresponding point of the web of a plate girder, and is equal to the sum of the weights situated between it and the centre of the girder. If W be the rate of loading per foot run, and y the distance of the bar from the centre, then the shearing strain equals $W \times y$. If N be the number of units of weight, upon all the apices, between any given vertical and the centre, and W the unit of weight, then $S = W \times \left(\frac{N-1}{2}\right)$. This rule applies to all the verticals except the last. The strain upon that bar is always equal to $\frac{W}{2}$

where W represents the total weight upon the whole girder.

Another useful fact to bear in mind is, that assuming the weight upon the central apex to be the unit, the strains upon the vertical bars lying between it and the end of the girder, follow the series of the odd numbers. The last bar is an exception to the rule, as will be seen. Thus, the strain upon the bar F F^1 is equal to $1 \cdot 25 \times 7 = 8 \cdot 75$ tons, that upon B $B^1 = 1 \cdot 25 \times 15 = 18 \cdot 75$ tons; but that upon A A^1 does not follow the law of the odd numbers, but is equal to $1 \cdot 25 \times 16 = 20 \cdot 0$ tons. Had there been no weight upon A A^1, but an equal weight of $2 \cdot 5$ upon every other apex, then the strains would have followed the law of the natural numbers. Similarly, the strains upon the diagonal bars are equal to those upon the verticals, resolved in the direction of their length. This may be easily shown. In Fig. 47, the strain upon the diagonal H K^1 equals by scale K^1 b $= 1 \cdot 43$ tons, and that upon the end diagonal equals by summation, $1 \cdot 43 \times 15 = 21 \cdot 45$ tons. This ought to equal the strain upon the vertical B B^1 resolved in the direction of A B^1. Let S^1 = strain upon A B^1, and S that upon B B^1, then by the resolution of right-angled triangles, putting θ for the inclination of the diagonal to the horizontal $S^1 = S \times \text{cosecant}\,\theta$. To find the value of θ we have tang. $\theta = \dfrac{B\,B^1}{A\,B} = \dfrac{8 \cdot 666}{5}$. and $\theta = 60°$.

Solving the equations, the strain $S^1 = 21 \cdot 64$ tons, which agrees near enough for all practical purposes with the former result. By a similar method of proceeding, the strains upon the flanges may be arrived at. Taking the upper flange for example, the value of $a\ b$ represents the strain resulting from the action of the weight at K,

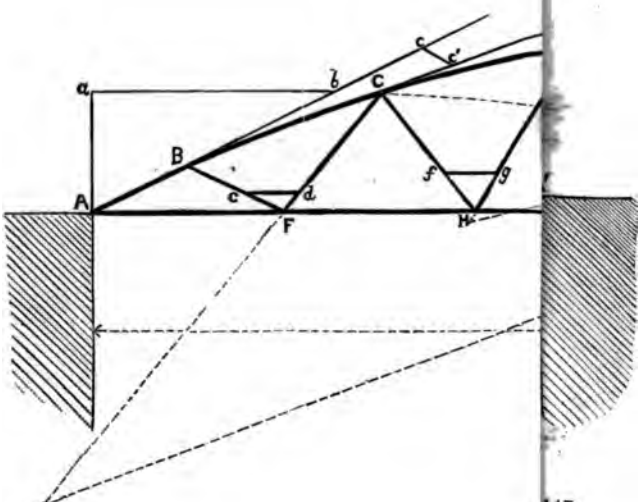
and $d\,e$ that due to the weight at H. The total strain upon the central bar H K of the flange is obtained by a summation of these strains. Thus the strain $a\,b = 0 \cdot 72$ tons is repeated by every pull of the diagonals on the apices, and as these pulls or tensile strains are all referred to the centre of the girder, the total strain resulting on the central bar H K from the action of the weight of $1 \cdot 25$ tons at K is equal to $8 \times 0 \cdot 72$ tons $= 5 \cdot 76$ tons. If the strain $d\,e = 1 \cdot 44$ tons, be now considered, it will be found that the number of times it is repeated by the action of all the weights at the apices, with the exception of that at A, and transferred to the central bar H K, is $7 + 6 + 5 + 4 + 3 + 2 + 1$. Consequently the strain upon H K, resulting from these weights is equal to $1 \cdot 44 \times (7 + 6 + 5 + 4 + 3 + 2 + 1) = 1 \cdot 44 \times 28 = 40 \cdot 32$ tons, and the total strain upon the central bar H K equals $5 \cdot 76 + 40 \cdot 32 = 46 \cdot 08$ tons. On referring to the former chapters, where the examples of the Warren and lattice girders are investigated, the agreement between the strains there calculated, and the present will be found to be very close. It follows from this investigation, that so long as certain data are maintained constant, the description of bracing employed has no influence whatever upon the strains upon the flanges. The question that will naturally present itself here for examination is—are the strains upon the flanges of a plate and lattice girder identical, so long as the loading, span, and depth are constant? In practice it is usually assumed that they are so, but there is no doubt that the continuous web of a plate girder does relieve the flanges of some portion of their strain. The advocates of the solid-sided principle have estimated this relief to be as much as one-sixth of the total strain, but this is doubtful.

Moreover, in the uncertain state of our knowledge, respecting the manner in which the strains do act in a continuous web, it would not be safe to lighten the flanges of a plate girder on the above supposition. The relation that exists between the web and the flanges with respect to their mutual straining forces, will be more clearly perceived when the bowstring girder is treated of, and other forms in which the flanges assume a curved shape.

It has been stated that the girder represented in Fig. 47, is not adapted for the support of any, but a fixed load situated at the centre. A little consideration will demonstrate this. Suppose the girder uniformly loaded, and let us take the weight at the apex C. This weight, by the laws of the lever, should be transferred to each abutment in portions, having an inverse ratio to their distance from those abutments. The weight compresses the vertical bar C C^1, but when it reaches the point C^1, there is only one tie C B, to receive it, so that it must all go to the near abutment. In order, therefore, to render the girder in Fig. 47 capable of supporting a uniformly distributed or movable load, the ties shown by the dotted lines must be inserted. This is not an economical method of construction. It is clear that when the web of a girder is constructed in this manner, there are three sets of bars required instead of two, as in the lattice system. It is optional in this kind of girder, to make the verticals act as ties and the diagonals as struts, or to make them all capable of acting both as struts and ties. When the design becomes complicated to this extent, it is difficult to determine precisely how the strains act. There are so many ways by which the load may be ultimately transferred to the abutment, that no particular one can be selected as the certain route.

As may be expected, there is a manifest waste of material in examples of this kind. Under certain conditions of loading, some of the bars undergo no strain, and as there are three sets of bars, one vertical and two diagonal, it is easy to see that they are never all undergoing strain at the same time. Moreover, it appears somewhat superfluous to introduce three sets of bars, when two are sufficient to answer every purpose of both a dead and a live load. With a weight only at C in Fig. 47, there will be no strain upon the bars $C D^1$ or $C B^1$. Generally speaking, when there are two series of diagonal bars, if we take a weight, for instance, at C only, there will be no strain upon any of the diagonal bars lying between that point and the centre of the girder, which slope downwards towards the centre. Neither will there be any strain upon the diagonal bars situated between the same point and the near abutment, which likewise slope downwards to that abutment. Under the conditions of a moving load, there are consequently continually some of the diagonals free from strain, the number depending altogether upon the position of the load. This is only what would be expected, from the somewhat similar circumstances attending the strains upon a lattice girder, when the load is of a variable character. If two designs be taken out in accordance with the requirements of the similar data of span, depth, and load, the lattice girder will be found to be more economical than any modifications of the example given in Fig. 47. There are one or two more specimens of the straight girder principle, but they are so rarely used that we shall not investigate them, more especially as they belong to an obsolete age of construction which has passed away. The class of girders that next claims our

attention is that in which one or both flanges are curved. Of these the purest type is the bow and string, but there are numerous combinations and crosses, as they might be termed, between it and the lattice proper, which will be subsequently investigated.



147.

CHAPTER XVII.

THE BOWSTRING GIRDER.

THE calculation of the strains upon the various parts of girders which have one or both of their flanges curved, is rather more complicated than that belonging to the class of structures we have been hitherto considering, in which both flanges are horizontal. This is chiefly owing to the continual alteration of the angle of inclination of the curved flange, as well as of the diagonal bars in the web. In addition, by reason of its form, the girder entrenches upon principles which do not prevail in the horizontal type of construction. It was found that the rule in straight girders, indicated a varying strain throughout the flanges, diminishing from a maximum at the centre to zero at the ends. The reverse took place in the web, except under special conditions of loading. With a uniformly distributed load, the diagonal bars underwent a minimum strain at the centre, equal to zero, and a maximum at the ends, the amount of the latter being invariably equal to half the total load upon the girder, multiplied by the cosecant of the angle of the inclination of the bar to the horizon. These conditions do not prevail in the class of girders we are about to investigate; in fact, they are almost completely reversed. In the flanges they become nearly uniformly equal throughout the whole length of the girder, and the maximum strains of compression and tension occur in the bars situated not near the ends, but at the centre of the span. In Fig. 48

is represented a bowstring girder, with a Warren, or single system of triangular bracing. The span is 40′, the depth at the centre 5 ft., and the rate of loading one ton per foot run upon the bottom flange. It is required to find the strains upon the flanges and web, under all conditions of loading. In this case it is better to take out the strains for each weight in succession; as the additions and subtraction of those of different sign, will determine the strains at once for a uniform load. When it is only necessary to ascertain the strains for a uniform loading, there is a readier and simpler means of arriving at them; but in the case of a large girder it would be necessary to follow the course we are about to adopt. The load at each apex, referring to the diagram, will consequently be equal to 5 tons, and, upon the principle of the lever, this will be transferred to each abutment, in portions in the inverse ratio of their corresponding distances. Let us commence with the 5 tons at the apex F′. Of this weight, $\frac{5 \times 7}{8} = 4 \cdot 375$ tons will be transferred to the abutment A′, and the rest, or 0·625 tons, conveyed to A. It is with this latter weight we are at present concerned, as it represents the vertical reaction at A. It is finally conveyed to the abutment by the portion of the upper flange represented by A B, and on arriving at A, is resisted by the whole of the lower flange in general, and by the last bar of it, A F, in particular. The first step is to produce all the separate bars A B, B C, C D, D E, E E′ of the upper flange to any convenient length, as shown in Fig. 48. Although, practically, the curve of the upper flange is an arc of a circle, yet for all the purposes of calculating the strains it is regarded as a polygon, the sides of which are obtained by joining, by a

straight line or cord, the respective apices of the triangles. The scale upon which Fig. 48 is drawn is almost too small to indicate the difference, but the left-hand half of the upper flange is shown as a polygon, while the right half is an arc of a circle. Where an approximate method, by the usual formula for horizontal girders, is employed for determining the strains upon the bow and the string, the upper curve is considered to be a parabola.

The weight of 0·625 tons being the vertical reaction at A, make A *a* equal to it upon any convenient scale, and draw *a b* to meet A B produced, *a b* and A *b* will equal the strains upon A F and A B, due to the portion of the weight at F′, which is conveyed to A. Before proceeding with the determination of the strains, Tables should be arranged, similar to Tables IX., X., and XI., for the purpose of tabulating them as they are progressively obtained from the diagram. The strains upon B C, B F, and F H are now required. Upon A B produced lay off B c = A b, draw c c' parallel to B F to meet B C produced; B c' is the strain upon B C, and c c' that upon the bar B F, the former being compressive and the latter tensile. If c c' be now laid off upon the bar B F, and c d drawn parallel to the lower flange, then c d will equal the strain upon F H, and F d that upon the bar F C. The total strain upon F H will equal a b + c d, and must be tabulated accordingly. The strains upon C D and C H may be obtained from those already found for B C and F C, and there are two methods of determining them. They may be deduced by taking the strains upon B C and F C separately, and resolving them in the directions of C D and C H, or by first finding the resultant of the former strains, and then completing the parallelogram of forces. In a former chapter the identity of these two methods,

which is self-evident, was shown in a diagram. To find the strains upon C D and C H, plot off upon B C produced, the length $C e' = B c'$, and from e' draw $e' f'$ parallel to $F c$ and equal $F d$. A line drawn from C to f' would represent the resultant of the strains in B C and F C. From the point f' draw $f g'$ parallel to C H, and $C g'$ represents the strain upon C D, and $f' g'$ that of tension upon C H. Making $H f = f' g'$, draw $f g$ parallel to the lower flange, and $f g$ gives the strain upon H K, and $H g$ that upon the bar H D. The total strain upon H K equals, consequently, $a b + c d + f g$. The determination of the strains upon the remaining bars, due to the reaction of a weight of 0·625 tons at A, is merely a repetition of the method already described, and is shown in the figure by the dotted lines. The action of the weight at F′ is now accounted for, and we may proceed to consider that of the next 5 tons placed at H′. The portion of this weight that is transferred to the abutment A is equal to 1·25 tons, or exactly double that resulting from the action of the weight at F′. The effect of its reaction at A will therefore be just double that of the former weight, and it only remains to double the strains already arrived at, and register them in the Tables. Similarly, for the weights situated at the apices K′ and L, all that is necessary is to multiply the strains already found for the weight at F′ by three and four, and we have those for these two other weights. No sooner do we come to the weights situated at the apices upon the other side of the centre of the girder, than this rule no longer holds for all the parts of the flanges and web. To consider the flanges first, and the weight at K. This weight, since its reaction at the abutment is five times that of the weight at F′, will affect the upper flange from A to E, and the lower

from A to K, to five times the extent of that weight, and the strains can be inserted in the Table, upon those parts of the flanges situated within these limits. The strains upon E E′ may, however, be readily arrived at by inspection, as it is evident that the weight at K, affects these parts of the girder to precisely the same extent as that at K′, the corresponding apex upon the opposite half of the girder. The strains upon the central portion E E′ of the upper flange may be at once written down, since the remaining strains from the weights H and F equal those already found for those at H′ and F′. All the strains upon the central part E E′ of the upper flange have been now calculated; it remains to determine those upon the remaining parts of both upper and lower flange, and upon the diagonals in the web. The effect of the weight at H has now to be considered. The strains upon A B, B C, and C D, are obtained, as already explained, by simply multiplying by six, the strains upon those members, due to the reaction of the weight at F′, and therefore there remains to be determined the strain upon D E, due to the weight at H. This strain is twice that due to the weight at F, so that if this be first determined, the other is also a known quantity. Draw E A′; from E lay off E a = 0·625 tons = the reaction at A′ of the weight at F. Draw $a\,a'$ horizontal, and $a'\,b'$ parallel to D E; then $a'\,b'$ is the strain upon D E from the weight at F, and twice this equals the strain required. It only remains now to account for the strains upon A B, B C, and C D, from the weight at F, to complete all the strains upon the upper flange. Those upon A B, B C, are respectively equal to seven times those found for the weight at F′; to obtain that at C D, we proceed as before for D E. Draw D A′; plot D p as before 0·625 tons; make $p\,p'$ horizon-

tal, and draw p' H parallel to C D, then p' H equals strain upon C D, which may be inserted in the Table, thus completing the strains upon the upper flange. The strains required upon the lower flange are those upon K L, due to the weights at K, H, and F; those upon H K and F H due to the weights at F and H; and that upon F H due to that at F. If we find the strain upon K L due to the weight at F, those due to the weights at H and K will be respectively twice and three times the amount. This strain upon K L, due to the load at F, is equal to the line $a\,a'$ in the diagram, and therefore the others are also known. Similarly, the line $p\,p'$ is equal to the strain upon H K from the load at F, and twice this is the strain upon the same part from the weight at H. Lastly, the strain upon F H, due to the weight at F, may be determined by drawing the resultant C A′ and proceeding as before. This last resolution of the forces is not given in the diagram, as it would confuse it too much; but it is precisely similar to that already performed at the apices D and E. The remaining strains upon the bars present no difficulty. Those brought upon E L by the weights upon one side of the centre of the girder, are equal to those produced by the weight upon the corresponding apices on the other half. To determine the strains upon the other bars it is only necessary to find that due to the weight at F, and the others are all multipliers of it. For the bar E K this strain is equal to E b'; and it may be found for all the others in a similar manner.

To ascertain the strains upon the various bars under the condition of a uniform load, it is only necessary to subtract the separate plus and minus strains, and the total will give the desired information. If a third Table be composed with the strains of the same character added together, then the maximum strain upon any bar

will be apparent, no matter what the condition of loading may be. This is shown in Table IX.

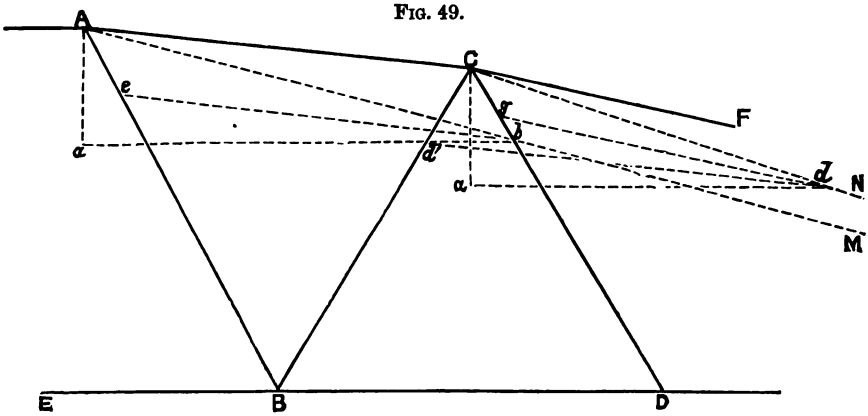
TABLE IX.

Bars.	Maximum compression.	Maximum tension.
B F	..	5·60
F C	2·31	5·70
C H	2·40	6·09
H D	4·05	6·30
D K	3·60	6·30
K E	3·92	6·60
E L	4·44	7·28

Instead of taking out the strains in the manner described in the diagram, by prolonging the various parts of the upper flange, they may be all equally arrived at by the other method, that is, by drawing lines from each apex to one of the abutments, plotting upon a vertical line that portion of any weight which is transferred to the abutment by that line, and resolving it in the directions of those parts of the girder which it affects. When this method is adopted, it is only necessary to follow the action of the weight at the apex F or F′, throughout the whole of the girder, as from it all the other strains can be determined. By this method the strains upon the parts of the upper flange are well checked, insomuch as the strain upon each separate part is twice determined, as will be seen from the diagram in Fig. 49. Suppose that we are following the action of that portion of the weight at F which is transferred to the abutment A′, and have arrived at any particular apex A. Let A M represent the direction of the resultant at A, or the line joining the apex A with the point A′ in Fig. 48. Make A a = that portion of the weight conveyed to the abutment at A′. Draw a b horizontal and b e parallel to A c, then a b equals the strain upon the E B; b e that

upon the top flange A C, and A e that upon the bar A B. The next point is to determine the strain due to the

Fig. 49.



same weight upon the upper flange C F, the lower B D, and the bars C B and C D. Make C a = A a, and proceed as before, first drawing C N to represent the line of transmitted pressure. Then a d equals the strain upon B D, d g that upon C F, C d' that upon C B, and C b that upon C D. But to obtain the strain c d' upon the bar C B, it is necessary to draw the line d d' parallel to A C. This line also represents the strain upon A C, and consequently $d\ d' = b\ e$. It is thus plain that knowing the vertical component of the strain at any apex, we can determine the strain upon the two bars meeting at that point, and also that upon the two separate parts of the upper flanges, also meeting the bars at the same apex. Another advantage of this method is that it gives the strain upon the different parts of the lower flange at once, without the necessity of adding continual increments to those already obtained. A reference to Fig. 48 and the accompanying Tables, demonstrates that under a uniform load, the strains upon the bars of the web are

very small, and also nearly uniform, if we except the end bar. The reason of the strain upon it being so much in excess of those upon the other bars, is due to the fact that it is only acted upon by a tensile strain, and has no counterbalancing and neutralizing strain of compression brought upon it, as is the case with the other bars. The maximum tension and compression upon any bar may be obtained at once by the diagram. Thus, to find the maximum compression upon the bar E L, all that is necessary is, to make E *a* equal the sum of those portions of the weights, situated at the apices between it and the abutment A, that are transferred to A′, and proceed, as already described. It can also be arrived at by estimation, for the sum of the weights acting upon the bar E L will equal $1+2+3$, multiplied by the transferred portions of the weight at F. Referring to Table XI., the compressive strain upon E L due to the weight at F is $0 \cdot 74$. The maximum compressive strain is, therefore, $1+2+3=6\times 0\cdot 74=4\cdot 44$ tons, which should tally with the result arrived at, by computing the same strain by the algebraical addition of all the separate ones upon the same bar. Although the method by mathematical analysis is not well suited to this description of girder, yet the principal calculations by the graphical process can be checked sufficiently readily to prove the identity of the two methods.

TABLE X.

Weight at	Parts of the Flange.								
	A B.	B C.	C D.	D E.	E E′.	A F.	F H.	H K.	K L.
F′	+ 1·38	+ 1·51	+ 1·70	+ 1·95	+ 2·45	− 1·22	− 1·48	− 1·83	− 2·35
H′	+ 2·76	+ 3·02	+ 3·40	+ 3·90	+ 4·90	− 2·44	− 2·96	− 3·66	− 4·70
K′	+ 4·14	+ 4·53	+ 5·10	+ 5·85	+ 7·35	− 3·66	− 4·44	− 5·49	− 7·05
L	+ 5·52	+ 6·04	+ 6·80	+ 7·80	+ 9·80	− 4·88	− 5·92	− 7·32	− 9·40
K	+ 6·90	+ 7·55	+ 8·50	+ 9·75	+ 7·35	− 6·10	− 7·40	− 9·15	− 8·45
H	+ 8·28	+ 9·06	+10·20	+ 6·80	+ 4·90	− 7·32	− 8·88	− 7·80	− 5·60
F	+ 9·66	+10·57	+ 5·10	+ 3·40	+ 2·45	− 8·54	− 6·45	− 3·90	− 2·80
Total..	+38·64	+42·28	+40·80	+39·45	+39·20	−34·16	−37·53	−39·15	−40·35

TABLE XI.

Weight at	Bars of the Web.						
	BF.	FC.	CH.	HD.	DK.	KE.	EL.
F'	−0·20	+0·11	−0·29	+0·27	−0·42	+0·40	−0·74
H'	−0·40	+0·22	−0·58	+0·54	−0·84	+0·80	−1·48
K'	−0·60	+0·33	−0·87	+0·81	−1·26	+1·12	−2·22
L	−0·80	+0·44	−1·16	+1·08	−1·68	+1·60	−2·84
K	−1·00	+0·55	−1·45	+1·35	−2·10	−3·30	+2·22
H	−1·20	+0·66	−1·74	−4·2	+2·4	−2·20	+1·40
F	−1·40	−5·70	+2·4	−2·1	+1·2	−1·10	+0·74
Total ..	−5·60	−3·39	−3·69	−2·25	−2·70	−2·68	−2·84

This investigation respecting the strains upon a bowstring girder, demonstrates that the result of curving the top flange of a girder, is to relieve the web of a portion of the shearing strain, which, in a horizontal girder, it resists alone. Consequently, if the curvature of the top flange be increased, until it assumes the form of a parabolic arc, it takes the whole shearing strain, and there is none whatever on the web. The curving of the upper flange, therefore, tends to produce a more equable distribution of strain upon it—that is, to increase the strains towards the ends. The strain upon the flanges at the centre of a bowstring girder, is the same as that upon those of a horizontal girder, and is given by the formula $S = \frac{W \times L}{8 \times D}$. Applying this to the last example, the strain upon the central part of the upper and lower flanges is equal to $\frac{40 \times 40}{8 \times 5} = 40$ tons. Upon referring to the Tables of strains, this will be found to agree very closely with the result arrived at by the graphical method. If the elevation of the girder were a perfect parabola, and the load uniformly distributed over it, the central strain would be constant throughout both the bow

and the string, or the upper and lower flanges, and there would thus be a very simple rule for determining the area of those parts. In the small example investigated, there would be no practical error introduced, by assuming the strains constant upon the bow and string, and therefore the sectional area would be constant likewise. A still nearer approach to the truth of this calculation will be afforded, in those examples in which the web consists simply of tie rods, which transmit the weight from the string to the bow. In this case there is a slight increase of strain towards the springing of the curve, which, in bridges of small span, may be neglected. Where the web consists solely of vertical rods, the strain upon each is exactly equal to the share of the load it bears, and is uniform upon all of them. There is thus a great difference between the manner in which the strains are transferred in a bowstring, and in a horizontal girder. In the latter the strains upon the bars of the web are augmentative, each bar nearer to the abutment having to sustain not only its own share of the load, but also those portions which are carried by its fellows and are transferred to it. With a load uniformly distributed over the girder, the starting or datum point of the transference commences at the centre; but in the case of a movable load, from the end of the segment covered by the load. The usual form of the web of a bowstring girder consists of vertical uprights, acting as suspension rods, and diagonal bracing, which is introduced for the purpose of stiffening it, and also to counterbalance the effect of a moving load. A preferable form is that in which there are no vertical bars whatever in the web, but in which the web is simply a diagonal lattice, the number of series of triangles depending upon the size of the girder. The same objection

that is to be found against the introduction of both vertical and diagonal bracing in horizontal girders, holds in the instance of those of the bowstring type. With a fixed load uniformly distributed there is no absolute necessity for any but vertical ties, and when the girder is under conditions similar to those obtaining in a railway bridge, the web can be more economically arranged, solely upon the lattice system.

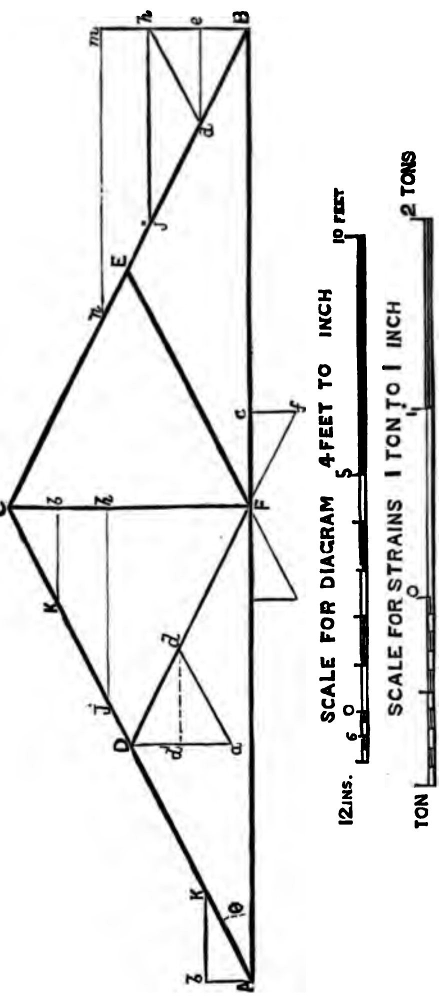
CHAPTER XVIII.

ROOF TRUSSES, SIMPLEST FORM.

THE great importance to an engineer of a thorough acquaintance with those forms of iron trussed girders, which are generally adopted in the case of roofs, is demonstrated by the increasing favour with which they are regarded by the profession, and their employment, on a large scale, for railway stations and buildings, in situations in which a lofty roof, combined with an unbroken area underneath, are indispensable qualifications. It is, however, not only in large examples that iron roofs are met with, but from the economical manner in which the bracing can be adjusted, to suit the strains brought upon the various bars, they are equally well adapted for very moderate spans. The simplest case is that represented in Fig. 50, and the strain upon the different members, will be ascertained and tabulated, in a manner similar to that employed for the girders in previous chapters. It is an ordinary king-roof principal, and answers well for small spans. In the figure the span is 20′, the rise 5′, and the total weight upon the whole principal is taken equal to two tons, so that one ton will be the load distributed over one-half of it. The same conditions will be assumed with respect to roofs as have been already laid down for bridges, and it will be always considered that the component parts of the structure are strained only in the direction of their length. The component parts of all the examples of roofs, which will be investigated, will be

assumed to be unacted upon by any others trains except those already mentioned, and will virtually be secured against their influence, either by their short length, bracing, or form of section. Before proceeding to the graphical analysis of the strains upon the diagram, the first step is to ascertain the manner in which the load is distributed. With one ton upon the half truss there will be $0 \cdot 25$ tons supported directly at A, by the reaction of the supports, and will practically exert no effect upon any part of the principal: $0 \cdot 25$ tons at the apex C, which is resisted by the counteraction of $0 \cdot 25$ tons, due to the load upon the other half of the truss, making in all $0 \cdot 5$ tons at the point C, and $0 \cdot 25 + 0 \cdot 25 = 0 \cdot 5$ at the point D. As we are only considering half the principal, the load at the apex C will be taken equal to

FIG. 50.



0·25 tons, or one-half of that which is supported at that point by the joint action of the two rafters. The distribution, therefore, will be as follows:—A quarter of a ton at A and C, and half a ton at D. Sometimes the whole load on one rafter is considered to be equally divided among all the points of supports, in which case there would be one-third of a ton at A, D, and C. The former method is to be preferred as the more accurate, and it will always be adhered to in all similar instances. Moreover, in making the assumption that the weight is equally distributed, there is a larger portion borne directly by the support at A than upon the former supposition; and as this is considered to exert no strain upon the truss, it should evidently be kept as small as possible. Let us now examine in detail the action of the weight of the several points upon the rafter, and determine the strains they give rise to in the various parts of the semi-truss.

The weight at A is resisted directly by the vertical reaction of the wall, and consequently produces no strain upon any part of the principal, so that we may pass on to that at D. This weight of 0·5 tons is in the first instance supported by the resistance of the lower part of the rafter A D, and that of the strut D F, causing strains of compression in both of them. Their amount may be readily determined. Make D *a* by scale equal to 0·5 tons; draw *a d* parallel to A D to meet the strut D F, and *a d* will equal the strain upon A D, and D *d* that upon D F. The strain A D upon the rafter, is transferred to the point A, where it is resisted by the action of the tie rod A F, and the vertical reaction of the support at A. Making A K equal to *a d*, and drawing K *b* parallel to the tie rod—that is, horizontal, to meet the

vertical line A b—the strain upon the tie rod A F is equal to b K. These, however, are not the only strains brought upon the part A D of the rafter and the tie rod A F by the action of the weight at D, as will be apparent on proceeding to examine into the effect of the strain D d upon the strut D F. The compressive strain D d is transferred to the point F, where it is resisted by the bars A F and F C. Plotting F f equal to D d, on the prolongation of the strut D F, and drawing f c parallel to F C, the strains upon A F and F C are represented by F f and F c. The strain upon A F may be disregarded, as it is counterbalanced by one of an opposite tendency and equal in amount from the weight at E, which also brings another equal strain upon the king rod. The total strain upon the king rod is equal to $2 \times f\,c$, but only half of this has to be regarded as affecting the other members of the half truss. Following the action of the strain f c, it is transferred to the point C, where it is represented by C b. If b f be drawn parallel to the tie rod A F, then C K will represent the strain upon the part C D of the rafter. This strain is again transferred to the point of support A, thereby causing an additional strain upon the lower part of the rafter A D, and upon the tie rod A F. So far, therefore, the total strain upon A D is equal to A K + C K = 2 A K, and that upon A F to 2 b K. In this instance the separate strains upon each member of the principal are equal, but this is partly due to the manner in which the load is distributed, the ratio between the span and rise of the roof, and the horizontality of the tie rod, as will be more fully perceived in checking the strains by mathematical analysis. The whole action of the weight at D has now been accounted for, and it remains to examine into that of the apex C.

This, by the distribution of the load, is equal to 0·25 tons, and, upon the scale of strains, equal to C *b*. Its action is therefore identical with that already considered, and it impresses upon the rafter an additional strain equal to C K upon C D and upon A D, and, consequently, an additional strain, equal to *b* K, upon the horizontal tie rod. Instead of taking the last two strains separately, they might have been made equal to C *h*, and, consequently, *c j* and *h j* would have represented the result upon the two parts of the rafter and the tie rod, being each of them respectively equal to 2 C K and 2 *b* K. A reference to the diagram will indicate that the strains may be arrived at in a somewhat different manner, by resolving the forces as shown at B. If these be compared, by means of a pair of dividers, with those already determined, the identity will be established. The strains may now be tabulated as represented in Table I., and may be briefly summed up as follows:— Strain upon A D = + 3 A K; upon D C = + 2 C K = C*j*; upon A F = − 3 *b* K; upon C F = − 2 *f c*; and upon D F = + D *d*.

There is clearly some analogy in the action of the strains upon a trussed roof and those upon a girder. In both instances, they are augmentative, according to the number of separate parts or bars in the structure, but the direction in which the increase takes place is not the same. Thus in a horizontal girder the strain upon the flanges increases towards the centre, but in a roof they increase towards the abutments, the lower end of the rafter having to resist the maximum strain. A similar increase attends the strains upon the tie rod, as will be pointed out when examples are treated of, in which the tie rod consists of two or more separate bars. If the

rafter C B be considered in the light of the last, or end bar, of a lattice or Warren girder, the total strain upon the lower portion may be arrived at in exactly the same manner as in that case. The total reaction at B is equal to 1 ton, but of this one quarter is directly supported by the wall, so that the portion affecting the rafter is reduced to 70·5 tons. Making B m = 0·75 tons, and drawing m n parallel to the tie rod, we obtain B n, equal to the total strain upon E B, and m n equal the total pull upon the tie rod. It has been assumed in this investigation that there is no weight, such as a floor, for instance, placed upon the tie rod, but if such should be the case, it should be distributed between the three points of support A, F, and B, and the weight added to the strain already obtained on F C. The result will be an increase in all the strains with the exception of that upon the struts D F and F E. In this particular description of iron structures there is very rarely any permanent load upon the tie rod. During the erection of the roof, and at subsequent periods of repair, the tie rod is subjected to a small permanent load, consisting of the necessary scaffolding and workmen, but this is not of sufficient importance to be taken into the calculation, as the margin allowed for safety will more than cover it.

Where there are so few parts, as in the first example we have selected in the diagram, all the strains may be readily calculated by a few simple equations, directly the theory of their action is understood, that is provided their effect upon the various members of the truss, can be traced from their origin to their final resistance at the points of support. Let W represent the total weight upon one-half of the truss, then, there will be a

weight of $\frac{W}{2}$ situated at the point D, and of $\frac{W}{4}$ at A and C. The strain upon the end of the rafter resulting from the weight $\frac{W}{2}$ is equal to $a\ d$, and by construction $a\ d = d$ D. Drawing $d\ d'$ parallel to the tie rod, $a\ d' = d'$ D $= \frac{W}{4}$ and angle $a\ d\ d' = \theta$. Putting $a\ d =$ S = strain on A D, we have $S = \frac{W}{4 \text{ sine } \theta} = \frac{W}{4} \text{ cosec. } \theta$. To find the value of θ, we put $\text{tang. } \theta = \frac{\text{C F}}{\text{A F}} = \frac{5}{10} = 0 \cdot 5000$ and $\theta =$ practically 26° 34′. Tracing the action of the weight $\frac{W}{4}$, which is the vertical component of that already determined, it will be seen that it is transferred to the apex C, and again resolved into a thrust upon the rafter. Moreover, there is another equal weight at C, which also brings a strain upon the whole rafter. Summing up, therefore, we have the total strain upon the lower part of the rafter equal to these three, and, therefore, S = $\left\{\frac{W}{4} + \frac{W}{4} + \frac{W}{4}\right\} \times \text{cosec. } \theta = \frac{3 \text{ W} \times \text{cosec. } \theta}{4} = 0 \cdot 75 \times \text{cosec. } \theta$.

But $\text{cosec. } \theta = \frac{1}{\text{sine } \theta^1}$, then $S = 0 \cdot 75 \times 2 \cdot 234 = 1 \cdot 67$ tons, which agrees with the result given in the Tables. The total tensile strain or pull upon the tie is the strain which resists this thrust on the rafter, and is consequently its horizontal component, and is represented by $m\ n$ in the diagram. By construction, the angle $m\ n$ B equals the angle θ, and putting S^1 for the pull on the tie we obtain $S^1 = S \times \text{cosine } \theta$. From above S = $0 \cdot 75 \times \text{cosec. } \theta$; therefore $S^1 = 0 \cdot 75 \times \text{cosec. } \theta \times$

cosine θ = $0 \cdot 75 \times$ cotang. $\theta = 0 \cdot 75 \times 2 = 1 \cdot 50$ tons. Obviously the thrust upon C D equals that upon A D, minus $a\ d$, equals therefore $S - (0 \cdot 25 \times \text{cosec.}\ \theta)$. The strain upon the strut D F = $a\ d$, and needs no further elucidation, and it only remains therefore to calculate that upon the king rod. This is equal to $2\ f\ c$. Let it be put equal to S^2, and we shall have the equation $S^2 = 2\ F\ f = 2\ D\ d = 2\ a\ d \times \text{sine}\ \theta$. But $a\ d = 0 \cdot 25 \times$ cosec. θ, therefore $S^2 = 2 \times 0 \cdot 25 \times \text{cosec.}\ \theta \times \text{sine}\ \theta = 2 \times 0 \cdot 25 = 0 \cdot 50$ tons. This completes the calculation of the strains upon the half truss. It must not be forgotten that a horizontal thrust is generated at the apex C, which is resisted by one similar in amount and direction, due to the action of the load upon the remaining half of the principal. This would be rendered apparent if the other half of the truss were replaced by a wall. As some of our readers may not be acquainted with trigonometrical calculations, the following equations will enable them to check some of the strains they have determined by the aid of the diagram by simple arithmetical means. The rule for the total strain upon the end of the rafter may be thus expressed in words: "The total strain upon the end of the rafter is equal to the total weight supported by it, multiplied by the length of the rafter, and divided by the rise of the roof." The rise of the roof is the distance from the middle part of the tie to the apex or junction of the rafters. If P be the length of the rafter, L the half span, and R the rise, then $P^2 = L^2 + R^2$ and

$$\begin{aligned} P &= \sqrt{L^2 + R^2} \\ &= \sqrt{100 + 25} = 11 \cdot 18. \end{aligned}$$

Substituting these values in the rule, the strain upon the end of the rafter equals $\dfrac{0 \cdot 75 \times 11 \cdot 18}{5} = 1 \cdot 67$ tons,

as before. Since the total strain upon the tie rod is the horizontal component of this, the rule for it is, "The total strain upon the tie rod is equal to the total weight upon the rafter multiplied by the half length of the tie, and divided by the rise." Consequently, in the present case it is equal to $\frac{0 \cdot 75 \times 10}{5} = 1 \cdot 50$ tons, as in Table XII. Similarly to a girder, the strains upon a roof

TABLE XII.

Weight at	Parts of the Half Truss.				
	AD.	DC.	AF.	CF.	DF.
A	..	..	..	..	..
D	+0·55 +0·55	+0·55 ..	−0·50 −0·50	−0·25 ..	+0·55 ..
C	+0·55	+0·55	−0·50	..	..
E	..	..	..	−0·25	..
Total	+1·65	+1·10	−1·50	−0·50	+0·55

principal are increased in the ratio of the span, and diminished in the inverse ratio of the rise, which is virtually the depth, and exercises the same influence over both examples of construction.

CHAPTER XIX.

ROOF TRUSS, WITH HORIZONTAL TIE ROD.

ALL iron roofs may be classed under one of two comprehensive heads. The first of these is that in which the rafter principle is the prominent feature, and the second embraces the many varieties of form in which this principle is not adopted. For very large spans the rafter principle is not so suitable as the other, which includes curved and circular roofs generally. The former may be divided into two chief kinds—one in which the tie rod is horizontal, and forms a straight line from the extremities of each rafter, and the other in which it is inclined upwards at any angle. In both instances the bracing may be arranged, or the truss "built up," in a variety of ways. There are at the same time certain advantages to be gained by a particular disposition of the struts and ties, which is the proper guide for the scientific and economical construction of trussed roofs. When the tie rod is horizontal, as represented in Fig. 51, there are two principal methods of arranging the bracing. That shown in the diagram is the most usual, but it is inferior to another, of which an example is given in the next chapter. The inclined bars in the truss shown in Fig. 51 are subject to a compressive strain, and the upright, or vertical, to one of tension. In addition to obtaining the strains upon a roof of this description by the graphic method, they can also be calculated by a few simple equations and formulæ, which are not applicable to

examples differing from that under consideration. The diagram in Fig. 51 represents the half skeleton elevation of a trussed or braced roof, with a span of 64′, and a rise of 14′ 6″. Each principal is supposed to be loaded

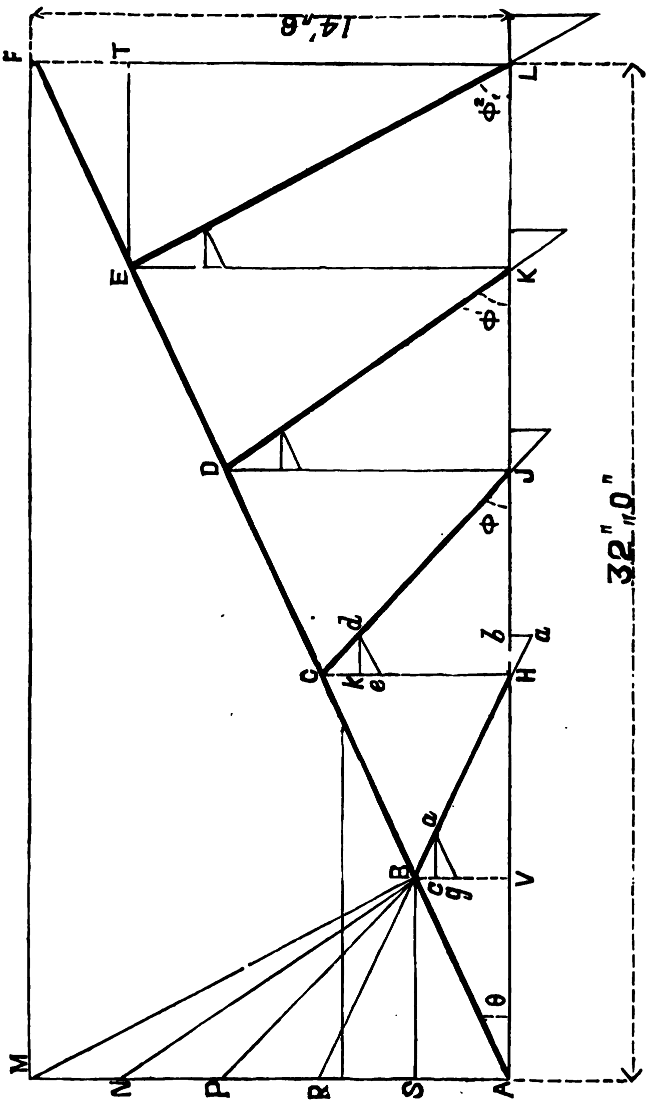


FIG. 51.

uniformly with a weight of 2 tons, or with 1 ton to each half truss. According to the rules already laid down for the distribution of loads upon roofs, the several apices of the triangles formed by the bracing will be loaded as follows:—At the apices B, C, D, E, the weight will equal 0·2 tons, while at A and F it will equal 0·1 tons. The total weight at F will be in reality equal to 0·2 tons, but only half of this will produce strain upon the half truss. As the graphic method of ascertaining the strains has already been demonstrated on a smaller scale, the successive steps will not be recapitulated, but the general outline only of the process given. The diagram, in which the strains are drawn to scale, and Table XIII. will enable the reader to follow the operation throughout. Commencing at B, the weight of 0·2 tons is plotted on the vertical line, from which a line is drawn parallel to the rafter to meet the strut B H, and the strains upon these two members at once determined. The strain upon the part of the tie A H may be obtained by either drawing a line from the intersection of the strains of the strut and rafter at *a* to meet the vertical already plotted, or by making H *a*, on the prolongation of the strut, equal to B *a*, and drawing *a b* vertically. H *b* will then also represent the strain upon the tie rod. Proceeding to the next vertex at C, make the vertical line equal to *a b* + the weight situated at that apex, that is equal to *a b* + 0·2 tons. Draw the parallels as before, and the strains upon the strut, rafters, and ties will be determined. A repetition of the same process will give the strains upon the remaining portions of the half roof, and they may then be summed up.

The total strain upon A B will equal that resulting from the weights at B, C, D, E, F, and will therefore be pro-

portional to the number of loaded apices. Similarly, the total strain upon C D will be equal to that upon A B, minus the strain resulting from the weight placed at B, and will therefore be also proportional to the number of loaded apices minus one, and so on for the remaining subdivisions of the rafter. All these strains are manifestly of a compressive nature, and tend to thrust out the point A, which thrust is resisted by the horizontal tie rod. The strains upon this latter member are also proportional to the number of loaded vertices in the truss, and may be found, like those upon the rafter, by direct calculation. In Table XIII. the strains upon the

TABLE XIII.

Parts of the half truss.	Weights at					Total strains.	
	B.	C.	D.	E.	F.		
A B	+0·25	+0·25	+0·25	+0·25	+1·18	+2·18	Rafter.
B C	..	+0·25	+0·25	+0·25	+1·18	+1·93	
C D	..	..	+0·25	+0·25	+1·18	+1·68	
D E	..	..	..	+0·25	+1·18	+1·43	
E F	..	..	..	..	+1·18	+1·18	
A H.. ..	−0·23	−0·23	−0·23	−0·23	−1·07	−1·99	Tie rod.
H J	..	−0·28	−0·23	−0·23	−1·07	−1·76	
J K	..	..	−0·23	−0·23	−1·07	−1·53	
K L	..	..	..	−0·23	−1·07	−1·30	
B H.. ..	+0·25	..	..	..	..	+0·25	Struts.
C J	..	+0·30	..	..	..	+0·30	
D K.. ..	..	..	+0·36	..	..	+0·36	
E L	..	..	..	+0·44	..	+0·44	
C H.. ..	−0·10	..	..	..	..	−0·10	Queen rods.
D J	..	..	−0·20	..	..	−0·20	
E K.. ..	..	..	..	−0·30	..	−0·30	
F L	..	..	..	..	−0·40	−0·80	King rod.

various parts of the truss, resulting from the action of the total weights at each vertex, are tabulated, and the Table shows that the strains upon the vertical ties, or queen rods, and upon the king rod, are likewise proportional to the number of loaded points. The total strain upon the king rod appears to be an exception to the

rule, but it is not so in reality, as only half this must be considered to belong to each half truss. The rule applies to every member of the half truss, with the exception of the struts B H, C J, D K, and E L. Their strains do not follow any particular law, and this is owing to the fact that their angle of inclination, both with the rafter and the tie rod, is continually altering. Having deduced the separate strains from the diagram, and tabulated them so as to obtain their sum, the maximum strains upon the rafter and tie may be now checked. In Table XIII. the weights at the apices C, D, E, and F, include the portion of the weights which is transferred to them, from each apex situated nearer the abutment than themselves. Putting S for the maximum strain upon the rafter, S_1 for that upon the tie rod, W for the total weight upon the half truss, and the remaining notation as in the last chapter, we have

$$S = \frac{W \times P}{R} = \frac{0 \cdot 9 \times 35 \cdot 33}{14 \cdot 5} = 2 \cdot 17 \text{ tons},$$

or, practically, the same result given in Table XIII. Similarly

$$S_1 = \frac{W \times L}{R} = \frac{0 \cdot 9 \times 32}{14 \cdot 5} = 1 \cdot 97.$$

In order to calculate all the strains—with the exception of those upon the inclined struts, which may be obtained by trigonometry, if desirable, instead of by a diagram—the following formulæ will be found applicable:—Let N equal the number of loaded apices in the half truss, which exert a strain upon the various parts of it, which will be one less than the actual number of points upon which a load is placed, since the point A, which is not strictly a vertex, is not included in the estimate, as the weight borne by it is resisted directly

by the upward reaction of the abutment, and exerts no strain upon the truss. The strains upon the other parts of the rafter may be thus found. The strain upon $BC = \frac{S(2N-1)}{2N}$, that upon $CD = \frac{S(N-1)}{N}$, that upon $DE = \frac{S(2N-3)}{2N}$, and that upon $EF = \frac{S(N-2)}{N}$. In like manner, if W be equal to the total load upon the semi-roof, the strains upon the verticals will be given in terms of it and N. The strain upon $CH = \frac{W}{2N}$, upon $DJ = \frac{W}{N}$, upon $EK = \frac{3W}{2N}$, and upon $FL = \frac{4W}{N}$. For the strains upon the ties the rule is the same as for those upon the rafters. Expressing the strains by the name of the bars

$$S^1 = AH,\ HJ = \frac{S^1(2N-1)}{2N},\ JK = \frac{S^1(N-1)}{N},\ KL = \frac{S^1(2N-3)}{2N}.$$

The diagram in Fig. 51 is drawn to a scale of 6′ to 1″, and the scale for strains is 1 ton to 1″. Whenever the angle of inclination of the rafter is given instead of the rise, it will be shorter to work by trigonometry to find S and S^1, and use the formulæ already mentioned, in which $S = \frac{W}{\sin. \theta}$ and $S^1 = \frac{W}{\text{tang.}\ \theta}$, in which θ is the angle of inclination of the rafter to the horizon.

The truss in the diagram may be regarded as composed of one principal and several secondary or subsidiary trusses, and there are consequently two sets of strains brought upon it; the one set being due to the action of the load upon the principal or primary triangle, and the

other resulting from the introduction of the smaller ones. Suppose for a moment that all these latter were removed, then the only strains upon the large triangle A F L would be a thrust upon the rafter A F, and a pull upon the tie A L. As there would be no intermediate points of support, the whole weight of W would have to be supported at the points A and F, and the truss would at once be reduced to its simplest form. It is known that the weight at the former point produces no strain upon any part of the truss, therefore the resultant strains upon the rafter and tie are due to the action of $\frac{W}{2}$ at the apex F. If A M be drawn equal and parallel to F L, and also made equal to $\frac{W}{2}$, the strains upon the rafter and the tie will be represented in amount by A F and M F. On referring to the Table, these will be found equal to those due to the action of the weight at F, thus proving the accuracy of the successive steps of the former process. The strains upon the other parts of the half roof may be now obtained without going through all the intermediate steps, since they are all multiples of one constant, with the exception of those upon the inclined struts. Draw B M. In the triangles A B M, E F L, the sides A B, A M are respectively equal to E F, F L, and the angle B A M equals the angle E F L. Consequently, the remaining sides and angles of the one triangle are equal to those of the other, and B M is parallel to E L. Draw the lines B N, B P, B R, and their lengths will be equal to those of the struts, and will represent the several strains upon them. It will be found that the intersection of the lines parallel to the struts with the vertical A M, together with a horizontal

line drawn from the point B, will divide A M into equal parts, which represent the load upon the half roof. The line A S equals the sum of the weights acting at A and F. The line B S will equal the constant strain for each bar of the tie rod, and A B that for each separate part of the rafter. The strains upon the vertical queen and king rods will be equal multiples of A S, in the order of their distance from the point A. The introduction of subsidiary, or secondary trusses, at once increases the strains upon the parts of the principal truss, but they are absolutely necessary to prevent the rafter and tie yielding by deflection. Table XIII. indicates that while they augment very considerably the original strain upon the primary truss, they are not subject, comparatively speaking, to much strain themselves. They act more as the carriers, or mediums of transferring strain, than as the absolute recipients of it, and the result of their action is nearly always to increase the original strain with respect to the member they transfer it to. Thus a total strain of 0·50 tons at F produces strains of the amount of 1·18 and 1·07 upon the rafter and tie. The strains upon the truss may now be calculated trigonometrically, and we will commence with those upon the inclined struts. In the diagram, since A V = V H, in the two triangles A V B, B V H, B V is common, the two right angles are equal, and the triangles are equal, and A B = B H, and the strain upon the strut due to a weight at B equals that upon the end of the rafter A B. Making W equal the load at B, θ the angle of inclination of the rafter, and S the strain upon the strut B H, the value of it is given by the equation $S = \frac{W \times \text{cosec.}\,\theta}{2}$. The angle θ is known, because the rise and half span are known, for calling

these respectively R and T, tang. $\theta = \frac{R}{T} = 24° \ 22' \ 30''$. From the distribution of the load the value of W at B is $0 \cdot 2$ tons, and in order to find the strain upon the next strut C J, we must ascertain the total pressure at C, which will consist of the original value of W, plus the portion transferred by the strut B H to the point H. This will be equal to B c or a b, which it is easy to prove is equal to $\frac{W}{2}$. The triangles B g a and A B H are similar to one another and equi-angular, and

$$Bc : cg :: AV : VH \therefore Bc = cg = \frac{W}{2}.$$

The total value for W therefore at C is known, and the calculation can be proceeded with. Referring to the diagram:—C d : C e :: sine C e d : sine C d e. But sine C e d = sine A C H = cos. θ, and sine C d e = sine A C J, since the sine of an angle equals the sine of its supplement. Again, by the construction, C d equals the strain upon the strut = S, and C e = W. Consequently, the equation now stands

$$S : W :: \cos.\ \theta : \text{sine } ACJ \text{ and } S = \frac{W \times \cos.\ \theta}{\text{sine } ACJ}.$$

Put angle A C J = θ^1, and $S = \frac{W \cos.\ \theta}{\text{sine } \theta^1}$. If the angle θ^1 were known, the strain upon C J could be determined. Make the angle A J C = ϕ, and we have the following proportion:—Sine θ^1 : sine ϕ : : A J : A C. But from the diagram $AJ = \frac{3 \times T}{5}$, and $AC = \frac{2 \times L}{5}$, putting L for the length of the rafter, which is a known quantity. Therefore sine $\theta^1 = \frac{\text{sine } \phi \times 3\ T}{2 \times L}$. By construction

$T = L \times \cos. \theta$, and the equation becomes

$$\text{sine } \theta^1 = \frac{\text{sine } \phi \times 3 \cos. \theta}{2}.$$

Substituting this value in the equation for S, we obtain $S = \dfrac{W \times 2}{3 \times \text{sine } \phi}$.

It now remains to find the value of sine ϕ. In the triangle C H J, tang. $\phi = \dfrac{CH}{HJ}$. But $CH = \dfrac{2R}{5}$, and $HJ = \dfrac{T}{5}$; therefore tang. $\phi = \dfrac{2R}{T}$, and, since tang. $\theta = \dfrac{R}{T}$, tang. $\phi = 2$ tang. θ. The angle ϕ is thus known, and from it, S can be calculated. Similarly for the strains upon the other struts, the values of ϕ^1 and ϕ^2 must be found, and it will be perceived that the tangents are respective multiples of that of θ. Thus,

$$\text{tang. } \phi^1 = \frac{DJ}{JK} = \frac{3R}{T} = 3 \text{ tang. } \theta,$$

and tang. $\phi^2 = \dfrac{4R}{T} = 4$ tang. θ. If the angles A D K, A E L, be made respectively equal to θ^2 and θ^3, the corresponding values of S will be $\dfrac{W \cos. \theta}{\text{sine } \theta^2}$; $\dfrac{W \cos. \theta}{\text{sine } \theta^3}$; or their equivalents $\dfrac{3 \times W}{4 \text{ sine } \phi^1}$ and $\dfrac{4 \times W}{5 \text{ sine } \phi^2}$.

Let us take an example and check the strains given in Table XIII. The strain upon the strut D K = 0·36 tons. By the formula, the strain for this strut equals $\dfrac{3 \times W}{4 \text{ sine } \phi^1}$, $W = 0 \cdot 4$ tons, and tang. $\phi^1 = 3$ tang. θ, from which $\phi^1 = 61° \, 7'$, and $S = 0 \cdot 345$ tons, which is a sufficiently close approximation to that arrived at in the diagram. The reason of the constancy of the strain, brought suc-

cessively upon the different parts of the rafter and tie rod, is evident upon an inspection of the diagram. The small triangles $a\ c\ g$, $d\ k\ e$; are equal to one another. Since $c\,g = \mathrm{K}\,e = \frac{\mathrm{W}}{2}$, the strain upon the strut is always equal to $\frac{\mathrm{W} \times \text{cosec.}\,\theta}{2}$. So also for the strains upon the tie, which are represented by $a\,c = d\,k = \frac{\mathrm{W} \times \text{cotang.}\,\theta}{2}$. Rules have already been given for finding the total or maximum strain upon the rafter and tie, so that those due to the pull of the king rod may be obtained by simple subtraction, or they may be readily calculated from the triangle E F T. It has been assumed that the value for W in the formulæ given for the strains upon the struts was known, but it may be determined as follows: Let ω equal the original weight placed upon each vertex, formed by the intersection of the rafter and a strut, and let N equal the number of the vertex counting from B. The value of W will be equal to $\omega + \frac{\mathrm{N}\,\omega}{2} = \omega\left(\frac{2 + \mathrm{N}}{2}\right)$. Thus the pressure at E will equal $0 \cdot 2\left(\frac{2+3}{2}\right) = 0 \cdot 5$ tons, as in the diagram. As the original weight at F is only half that on the other apices, it will form an exception to this rule. An objection to this form of roof is that as the angle of inclination of the struts is always varying, they are not alike under favourable circumstances for resisting strain. The last strut, for instance B H, is placed at an angle of about twenty-five degrees with the tie, which is a very unfavourable position. A better form of truss is obtained by placing the struts as nearly perpendicular to the rafter as the construction and design will permit.

CHAPTER XX.

SINGLE ROOF TRUSS WITH DIAGONAL BRACING.

It has been already mentioned that the tie rod of a roof truss is usually placed in an inclined position. Before proceeding to analyze an example of that description, we will investigate the one illustrated in Fig. 52, in which the span, rise, and rate of loading are identical with those adopted in the example in the last chapter. The difference consists in the manner in which the bracing is arranged, and the two examples should be carefully studied and compared together, so that an accurate estimate may be formed of their relative advantages. In the diagram Fig. 52, the struts approach nearer to a perpendicular position with respect to the rafter, while the ties or queen rods, are no longer vertical, but inclined at different angles to the horizontal. This description of bracing, allowing for the varying angles of inclination, is similar to that of a Warren truss or girder, and it is preferable to the older, and somewhat stereotyped form represented in Fig. 51. The majority of the struts in the latter instance are shorter than the corresponding ones in the former figure, and are consequently not strained to quite the same amount. This difference would be shown more conclusively if, instead of the unit weight selected for the purpose, the actual weight likely to be placed upon a roof of the given dimensions had been adopted. The former, however, possesses the

advantage of acting as a standard for all similarly trussed roofs of the same span and rise, since, to find the actual strains upon the various parts, it is sufficient to multiply the strains given in the tables by the proper constant, or the number representing the ratio between the unit adopted and the load in question. In the present example the strains are divided into two classes, under the heads of direct and transmitted strains. So that it will be perceived how the action of the same weight is multiplied, again and again, by the different parts of the bracing. It is of very little use to be acquainted with the total strain on any particular bar, unless the designer of a structure is capable of analyzing that strain, dividing, and, as it were, dissecting it into every one of its component parts. Mathematical formulæ, wherever they are applicable, suffice for calculating the total amount of any strain, but they do not afford the slightest clue to the manner in which it has been gradually and successively accumulated. They give, it is true, the result of a load, but impart no information respecting its intermediate action. As an instance, take the formula for the strain upon the last bar of a Warren girder, where S equals the strain, W the total load, and θ the angle of inclination of the bars to the horizon. The calculation is at once made by the equation $S = \frac{W \times \text{cosec.}\,\theta}{2}$; but the equation affords not the slightest information, respecting the manner in which the successive strains are obtained, until they reach the total represented by S.

The direct strains result from the action of those portions of the total load situated at the points B, C, D, E, and F, and affect the several bars of the rafter, and the inclined struts attached to them. It will not be neces-

sary to do more than indicate briefly the effect of the strains, represented by the lines of the diagram, as the principles of the analysis and the geometrical reasoning have already been enunciated. Moreover, Table XIV. has been constructed to show, at a glance, the direct strains resulting from the weights placed at the different

TABLE XIV.

Parts of the Half Truss.	Direct weights at B	C	D	E	F	Total Strains.	
A B	+0·165	+0·0975	+0·0675	+0·055	+0·245	+0·630	Rafter.
B C	..	+0·0975	+0·0675	+0·055	+0·245	+0·465	
C D	..	..	+0·0675	+0·055	+0·245	+0·367	
D E	..	..	..	+0·055	+0·245	+0·300	
E F	..	..	..	..	+0·245	+0·245	
B H	+0·200	..	..	..	..	+0·200	Struts.
C J	..	+0·185	..	..	..	+0·185	
D K	..	..	+0·182	..	..	+0·182	
E L	..	..	..	+0·185	..	+0·185	

apices of the triangles. The distribution of the load will be the same as before; therefore making the vertical lines B a, C d, D e, E f, respectively equal to 0·2 tons, the direct strains upon the rafter and struts are given by the other sides of the triangles, and will be found to agree with those in Table XIV. At the apex F the weight is only half that situated at the lower apices; so that F j = 0·1, and F k equals the strain upon the part of the rafter E F, and is also added to those already obtained for the other parts. It might perhaps be considered that the addition of the strain upon B C, or upon any other bar of the rafter to that upon A B, should be regarded as a transmitted strain, and not a direct one. So it would, were the direction of the strain altered, but it is not; both the direction and nature of the strain remain constant; and, moreover, A B, B C, and in fact, all the

TABLE

PARTS OF THE

Weights at	A B	B C	C D	D E	E F	A H	H J	J K
	+0·120	+0·120	+0·050	+0·032	+0·095	−0·210	−0·067	−0·030
	+0·050	+0·050	+0·032	+0·095	..	−0·067	−0·030	−0·020
B	+0·032	+0·032	+0·095	..	..	−0·030	−0·020	−0·073
	+0·095	+0·095	..	..	..	−0·020	−0·073	
	..	..	..	..	..	−0·073		
	+0·115	+0·115	+0·115	+0·060	+0·200	−0·143	−0·143	−0·070
C	+0·060	+0·060	+0·060	+0·200	..	−0·070	−0·070	−0·037
	+0·200	+0·200	+0·200	..	..	−0·037	−0·037	−0·160
	..	..	..	..	..	−0·160	−0·160	
	+0·087	+0·087	+0·087	+0·087	+0·305	−0·120	−0·120	−0·120
D	+0·305	+0·305	+0·305	+0·305	..	−0·075	−0·073	−0·073
	..	..	..	..	..	−0·275	−0·273	−0·273
E	+0·425	+0·425	+0·425	+0·425	+0·425	−0·090	−0·090	−0·090
	..	..	..	..	..	−0·340	−0·340	−0·340
F	..	..	..	..	..	−0·223	−0·223	−0·223
Total	+1·499	+1·499	+1·379	+1·329	+1·297	−1·929	−1·719	−1·509

separate bars of the rafter A F, are in reality but one bar, although theoretically subdivided. This is clearly not the case with the strain induced upon the bar H C, by the action of the weight at B. The compressive strain in the strut B H, is changed both in direction and character, when transmitted to the bar H C, or A H, but the strain upon the bar B C from the weight at C, undergoes no change of any kind in amount, character, or direction in passing to the bar A B. It is a simple addition, and so for the other strains transferred from C D, D E, and E F.

There is no readier method of ascertaining the strains in Table XIV. than that demonstrated in the diagram. In consequence of the inclination of the bars and their deviation from the vertical, the trigonometrical calculation of the thrusts, or compressive strains upon the different parts of the rafter and struts, is not capable of being so easily effected as in the former instance, where the queen rods were perpendicular, nor is there any advantage to be gained in resorting to that method. The manner in which the half truss is affected by the transmitted strains is represented in Table XV. By the aid of the diagram

XV.

HALF TRUSS.

KL	LM	BH	CJ	DK	EL	HC	JD	KE	LF
−0·020 −0·073	−0·073	..	+0·087	+0·050	+0·048	−0·150	−0·070	−0·048	−0·037
−0·037 −0·160	−0·160	..	..	+0·120	+0·075	..	−0·170	−0·100	−0·075
−0·073 −0·273	−0·273	..	..	..	+0·148	..	..	−0·180	−0·147
−0·090 −0·340 −0·223	−0·340 −0·223	..	..	..	..	..	..	..	−0·187
−1·289	−1·069	..	+0·087	+0·170	+0·271	−0·150	−0·240	−0·338	−0·446

there will be no difficulty in following the analysis, and there is no point calling for especial notice, with perhaps the exception of the strain upon the centre bar L M of the horizontal tie rod. This is found equal to −1·069. It is evident, on inspection, that the bar L M is not in any way affected by the strains upon the intermediate struts and ties, forming the component parts of the truss. The strain upon it is exactly the same as if they were all removed, and the truss consisted simply of a rafter A F and the half tie rod A M. The total load will then be supported at the two points A and F, half at each point. Make A N equal to this load equal to 0·5 tons, and draw N B parallel to A M. The line N B will scale 1·069, and will represent the strain upon the bar A M, or L M. Whatever form of truss may be adopted, or whatever may be the number of the secondary or subsidiary trusses, the strain upon the centre bar of a horizontal tie rod will be that due solely to the loading upon the primary truss, and will be altogether unaffected by the introduction of smaller secondary trusses and bracing. This will be better seen in the example of a roof with an inclined tie, as will also

several other conditions of strain, which are not so apparent in the simple instance in Fig. 52. The direct and transmitted strains may now be summed up, and tabulated as shown in Table XVI. The sum of the two

TABLE XVI.

Parts of the Half Truss.	Direct Strains.	Transmitted Strains.	Total Strains.	
A B	+0·630	+1·499	+2·129	Rafters.
B C	+0·465	+1·499	+1·964	
C D	+0·367	+1·379	+1·746	
D E	+0·300	+1·329	+1·629	
E F	+0·245	+1·297	+1·542	
A H	..	−1·929	−1·929	Tie rod.
H J	..	−1·719	−1·719	
J K	..	−1·509	−1·509	
K L	..	−1·289	−1·289	
L M	..	−1·069	−1·069	
B H	+0·200	..	+0·200	Struts.
C J	+0·185	+0·087	+0·272	
D K	+0·182	+0·170	+0·352	
E L	+0·185	+0·271	+0·456	
H C	..	−0·150	−0·150	Ties or Queen rods.
J D	..	−0·240	−0·240	
K E	..	−0·338	−0·338	
L F	..	−0·446	−0·446	

descriptions of strains, represents the total strain resulting from the whole weight of the truss. The strains upon the ends of the rafter and the tie rods, that is, upon the bars A B and A H, may be checked by plotting the total reaction of the load at the abutment, and completing the triangle of forces. Make A P equal the reaction, draw $P P^1$ parallel to the horizontal tie, and $A P^1$ and $P P^1$ will give the measure of the strains upon A B and A H to the same scale. Or the same results may be obtained by the formula already given, which, however, it must be remembered, only applies to those examples in which the tie rod is uniformly horizontal. Let S and S^1 equal respectively the strains upon the ends of the rafter and tie rod, or upon the bars A B and A H. Putting θ for the angle

of inclination F A M, and W for the total weight upon the half roof, then S $= W \times$ cosec. θ and $S^1 = W \times$ cot. θ and S $= 2 \cdot 124$, and $S^1 = 1 \cdot 929$, which agree with

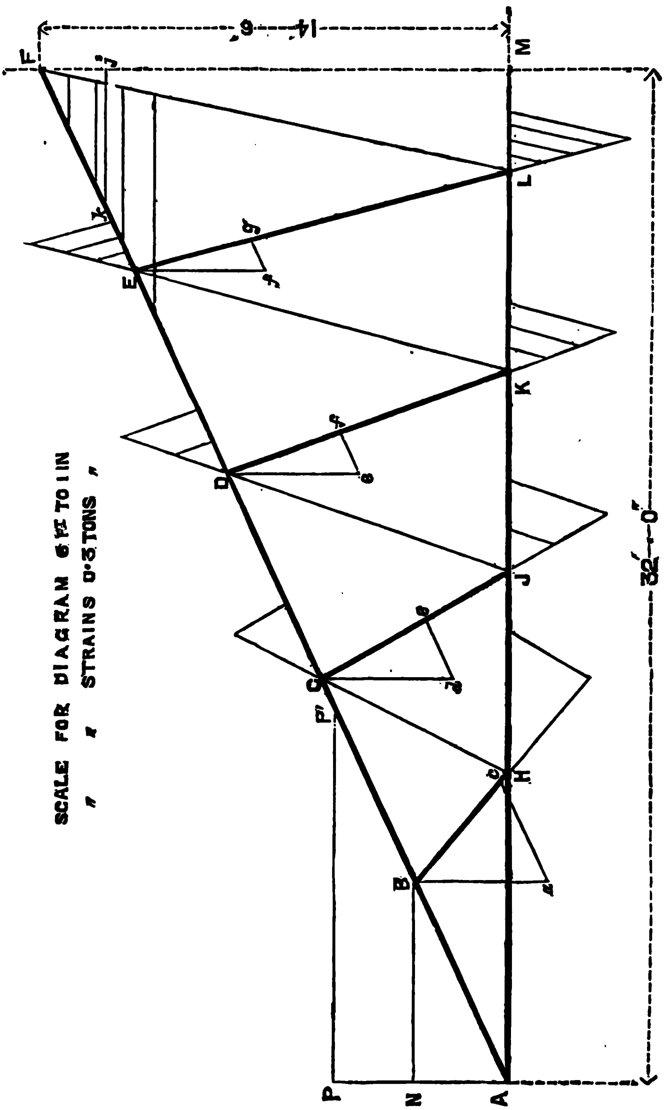


Fig. 52.

the strains found by summation in Table XVI. Similarly the strain upon the bar L M of the tie rod may be found by calculation. The natural cotangent of 25 deg. being 2·144, the strain required equals $2 \cdot 144 \times 0 \cdot 5 = 1 \cdot 07$ tons. The member which has the greatest influence upon the strains upon a roof is the tie rod. Directly this becomes inclined from the horizontal, it modifies the amount of the strains in all the component parts of the truss, and it is no longer possible to check the sums of the strains upon the ends of the rafter and the tie rod, by the same simple methods already adopted. This follows from the fact, that if the portion of the tie rod situated next to the rafter, be inclined upwards from the horizontal, while the central portion remains horizontal, there are no longer three forces making equilibrium at the abutment, but four. One operation is, therefore, not sufficient to resolve the strains upon all the bars affected by the vertical reaction at that point. It must not be assumed that the process of analysis, which answers for a simple example, is also applicable to others of a more complicated and scientific form.

In the practical designing of roofs, if they be thoroughly well secured by wind-ties and bracing from the sudden action of violent strains, the material may be taxed a little more than in the case of a bridge. So far as the parts in tension are concerned, it might be safe to increase the stereotyped five tons to six tons per inch of sectional area, but it would scarcely be prudent to adopt the same course with the parts in compression. The struts constitute the weak part of a roof truss, and there is, moreover, this important difference between it and a lattice bridge—the failure of one bar in the former would be certain to seriously

jeopardize, and probably destroy, the security of the others. This contingency is a well-known and a well-founded objection against the employment of the Warren girder, for any, except limited spans, whereas the fracture of one of the bars in the web of a lattice girder would affect that bar only.

CHAPTER XXI.

DOUBLE TRUSS, WITH INCLINED TIE ROD.

THE tie rod of a roof has hitherto been regarded as occupying a horizontal position, from the extremity of one rafter to that of the other, and a truss of this description answers well enough for spans of limited dimensions, and also in instances where the engineer is not troubled about the question of headway. Frequently this is the very question he has to deal with. To increase the headway, the obvious plan is to raise the tie rod. But since the strength of any single truss or girder is directly as the depth, the raising of the tie rod diminishes the depth, and therefore the strength of the truss; or, what amounts to the same, the strain upon the various members of the roof is increased. But this is of comparatively little consequence with other and more important considerations. There are certain given conditions which must be fulfilled, no matter what the strain may be, and the engineer has only to make the best of them under the circumstances. Supposing, therefore, that it is necessary to employ a description of truss, with the tie rod raised above the level of the extremities of the rafters, there are some points of difference existing between the two types which demand notice. A more correct distinction might be made, by calling one the single, and the other the double truss system, as a reference to Fig. 53 will indicate. The whole roof represented in the diagram

consists of two separate trusses A D C, B D C, which are united at the apex C, and held together by the horizontal tie rod D D. In the diagram, the parts in compression are shown by the thick, and those in tension, by the thin lines. The only point of identity that exists between the double and the single trussed roof, is in the king rod C E, which has no strain whatever on it provided two conditions are fulfilled. These are that the portion of the tie rod which is connected with it should be horizontal, and not sufficiently long to be liable to sag from its own weight. It might be imagined that, as the horizontal tie rod D D, prevents the feet of the separate rafters from being thrust outwards, it virtually has a strain upon it equal to the horizontal thrust; but such is not the case, and the error must be carefully guarded against. If the tie rod D D were directly attached to the extremities of each individual rafter, it would then be in the position of that belonging to the single truss system, and the pull upon the portion of it next to the rafter would equal the horizontal thrust of the roof. But in the present instance the pull upon it, due to the thrust of the rafter, can only be transferred to it through the medium of the inclined tie A D, which consequently alters both the direction and amount of the original strain. The strains upon the trusses themselves are dependent, both upon the pitch of the rafter, and the angles F A D, F B D, of the inclined tie rods, supposing span and load to be the same. Both these are also dependent upon the absolute pitch of the roof, that is, the angle C A B. There is a particular value for this angle, which causes the strain upon the bar A D to be exactly double that on D C. The advantage of this in practice is obvious, as it simplifies the number of independent

parts; since, whatever may be the scantling of D C, it is only necessary to use two bars instead of one, to obtain the requisite quantity of material in A D.

The reduction of the component parts of a structure, to as few dissimilar pieces as possible, is a consideration the importance of which cannot be over-estimated. This becomes imperative when the structure is intended to be erected in a foreign country, where skilled labour is scarce and dear, and sometimes not to be procured at any cost. A girder or roof, every component part of which was interchangeable, would be the perfection of simplicity, so far as erection in a foreign and distant country was concerned. There is no reason why in numerous examples this might not be attained, and a considerably nearer approach to so desirable a result, might be gained than that which now prevails, in larger and more pretentious designs. A glance at a good many of the structures dispatched to our colonies and dependencies, is sufficient to induce one to almost believe that complexity, not simplicity, was the aim of the designer, and that they considered the merit of the work, to consist in the multiplicity of its joints and articulations. An iron roof is a particularly favourable specimen of construction, to erect in a country where there is a scarcity of skilled labour, as the connection of its various parts can be accomplished through the medium of pins or bolts, and riveting is thus avoided. It is true that the junction of the web and flanges of iron girders, in bridges, is also effected by the use of pins, but in the latter case, it is not only optional, but preferable, to use rivets.

Before proceeding to analyze by diagram, the nature and amount of the strains, upon the double trussed roof represented in Fig. 53, a few of them may be ascertained

by calculation, and will thus serve as a check upon the other method. Put S for the span, R for the rise or depth of truss from C to E, L for the length of the rafters, W for the total load in tons upon the whole principal, and θ for the angle of the pitch of the roof. The distribution of the load on the half truss in reference to Fig. 53 will be $\frac{W}{4}$ at the point F, and $\frac{W}{8}$ at A and C. The total weight at the apex C will be $\frac{W}{4}$, but $\frac{W}{8}$ is all that concerns the strains upon one-half of the truss. To find the strain first on the strut F D, put S for the strain, and it becomes $S = \frac{W}{4} \times \cos.\ \theta$. If we take $W = 2$ tons, which makes the load on the half truss equal to unity, and $\theta = 26$ deg., we have $S = 0 \cdot 449$ tons. To determine the strains upon the different parts of the rafter, make the angle F A D $= \theta'$. Both these angles θ and θ' can be readily calculated, as the one is a function of the rise and span of the roof, and the other of the length of the rafter and the length of the strut F D, which is known by construction. Altogether there are three strains brought upon the rafter A C, which affect the portion A F, and two which affect F C. Calling these S_1, S_2, and S_3 respectively, we have their respective values

$$S_1 = \frac{W}{4} \times \text{tang.}\ \theta;\ S_2 = \frac{W}{4} \times \cos.\ \theta \times \cot.\ \theta';$$

$$\text{and } S_3 = \frac{W}{8} \times \frac{\text{sine } 90 + (\theta - \theta')}{\text{sine } \theta'}.$$

The part of the rafter F C is obviously not directly affected by the weight at F, which produces the strain S_1; therefore the strain upon F C will be equal to

$$(S_2 + S_3) = \frac{W}{4} \times \cos.\theta \times \cot.\theta' + \frac{W}{8} \times \frac{\text{sine}\,\{90 + (\theta - \theta')\}}{\text{sine}\,\theta'},$$

and that upon

$$A\,F = (S_1 + S_2 + S_3) = \frac{W}{4}(\text{tang.}\,\theta + \cos.\theta \times \cot.\theta') +$$
$$\frac{W}{8} \times \frac{\text{sine}\,\{90 + (\theta - \theta')\}}{\text{sine}\,\theta'}.$$

The formula may be put in another form, for let $(S_1 + S_2 + S_3) = M$, then

$$M = \frac{W}{8\,\text{sine}\,\theta'} \times \{(2\,\text{sine}\,\theta'\,\text{tang.}\,\theta + \cos.\theta\,\cot.\theta') + \text{sine}\,\{90 + (\theta - \theta')\}\}.$$

Substituting in this equation the correct values for the quantities we obtain

$$M = \frac{1}{4 \times 0{\cdot}258} \times \{(0{\cdot}516\,(0{\cdot}487 + 3{\cdot}349) + 0{\cdot}981)\} = 2{\cdot}86 \text{ tons.}$$

The strain upon F C can be obtained either from the formula given above, or more simply by subtracting from the last. Calling it N, we have $N = (M - S^1)$, $= (2{\cdot}86 - 0{\cdot}243) = 2{\cdot}617$ tons. A comparison should be made between these results, and those obtained for the strains upon the rafter, when the tie rod is horizontal, in order to trace the manner in which the inclination of the ties affects them. The angle θ' becomes an element in the calculation, and assists in complicating it. We may now ascertain the strains upon the inclined tie rods, A D and D C. There will be only one upon D C, due to the direct action of the weight at F, which will produce equal strains upon A D and D C. These may each be calculated from the formula

$$S_4 = \frac{S \times \cos.\theta'}{\text{sine}\,2\,\theta'} = \frac{S}{2\,\text{sine}\,\theta'} = 0{\cdot}87 \text{ tons.}$$

As this strain is transferred to the apex C, it is multiplied again on the rafter and the tie A D, which also receives an additional strain from the weight directly

superimposed at C. Therefore the total strain upon A D is equal to 2 $S_4 + S_5$, but S_5 is equal to $\frac{W}{8} \times \frac{\cos.\ \theta}{\text{sine}\ \theta'}$, and may be easily shown to be equal to S_4. For

$$S_4 = \frac{S \times \cos.\ \theta'}{\text{sine}\ 2\ \theta'} = \frac{W}{4} \times \cos.\ \theta \times \frac{\cos.\ \theta'}{\text{sine}\ 2\ \theta'}.$$

Substituting for the expression sine 2 θ' its equivalent 2 sine θ' cos. θ', the identity between the two equations is established, and the total strain upon A D becomes equal to 2·61 tons. But there is another strain upon D C due to a part of the strain upon A D. Let the portion of the strain upon A D equal S_6, that affects D C and D E. Then the additional strain upon D C will be given by the formula

$$S_7 = S_6 \times \frac{\text{sine}\ (\theta - \theta')}{\text{sine}\ (\theta + \theta')}.$$

Substituting in this equation the proper values, we find

$$S_7 = \frac{1 \cdot 73 \times \text{sine}\ 11^\circ}{\text{sine}\ 41^\circ} = 0 \cdot 50\ \text{tons};$$

thus making the total strain upon the tie D C equal to 1·37 tons. It only remains now to find the strain upon D E, which is found from the equation

$$S_8 = \frac{S_7 \times \text{sine}\ 2\ \theta'}{\text{sine}\ (\theta - \theta')}\ \text{or}\ S_7 = \frac{0 \cdot 50 \times \text{sine}\ 30^\circ}{\text{sine}\ 11^\circ} = 1 \cdot 31\ \text{tons}.$$

These calculations will be found to check sufficiently closely with those arrived at by the other methods, represented in Fig. 54, to prove the accuracy of the results for all practical purposes. The strain upon D E is the same as that of the horizontal thrust, modified by the action of the tie rod A D, for D E might be replaced by an abutment or buttress at the points A and B, without altering the conditions of equilibrium existing in the roof.

The diagram in Fig. 54 shows the lines necessary to obtain the strains upon the different parts of the truss, and in Table XVII. results are given so that they may be compared with those already obtained by calculation.

TABLE XVII.

Parts of Truss.	Weight at			Total Strains.	Remarks.
	A	F	C		
A F	0	{+0·225, +1·700}	+0·950	+2·875	Rafter.
F C	0	+1·700	+0·950	+2·650	Rafter.
F D	0	+0·450	..	+0·450	Strut.
A D	0	−1·750	−0·875	−2·625	Ties.
D C	0	−0·875	−0·500	−1·375	Ties.
D E	0	−0·670	−0·670	−1·340	Tie rod.

In the diagram there are two methods demonstrated, one showing the actual transference of the separate strains, and the other total strains upon the different members of the truss. The scale of the diagram is a quarter of an inch to the foot, and for the strains, of which the directions and amount are shown inside the truss, 1″ to the ton. For those shown by the dotted lines outside the truss, the scale is 2½″ to the ton. According to the distribution of load which is adopted, the total load upon the half principal being 1 ton, the load upon point F is 0·5 tons, and at A and C 0·25 tons respectively. The lines which indicate corresponding strains, are distinguished in the two methods as far as possible by the same letters, with the addition of dotting those belonging to the outside diagram of strains. Any line parallel to any given bar is a measure of the strain, or a part of the strain, upon it. The difference between the two methods is that the one, or successive method, gives the separate strains, brought by each weight upon the different parts of the truss, while the other method does not. Take, for

instance, the strain upon the two parts of the rafter A F, F C. By the former method the strain upon A F is ascertained by measuring the lines *a b*, *f* C, and *h* C, and that upon F C by *f* C and *h* C. By the latter the strain upon A F is equal to A C + A F, but the strain upon F C is equal to A C + b' F, the exact reason for which does not appear, as the manner in which the strains act is not investigated throughout. It is not the result alone that must be considered, but the means by which that result is obtained.

It is the preliminary steps which are the most important, and the very points which require accurate elucidation. In Fig. 55, the same truss is shown with the strains indicated by the lines, bearing the same letters as in Fig. 54, and the diagram is drawn in accordance with the method known as the "polygon of forces," first employed by the late Professor Rankine. While the results are perfectly accurate, the method fails, like the other, to trace the action of the strains, and can therefore supply no information, except to those who have already mastered the whole subject. The scale for the diagram is 2″ to the ton, so that the length of the lines can be at once compared with the sum of those in Fig. 54, which answers to the corresponding strains. A comparison of these two diagrams will point out that they agree not only in the total, but in the separate strains, much more closely than might be imagined. For example, the total strain upon the tie rod A D is equal in Fig. 54 to $c\,d + f\,e + h\,g = 3\,c\,d$. On referring to Fig. 55, it will be seen that these separate strains are correspondingly represented by the three subdivisions of the line A D. Similarly for the strains upon D C, which are equal in Fig. 54 to D d + K l, and in Fig. 55 the subdivisions of the line

D C. An examination of the method of the "polygon of forces" will demonstrate that it is in every way superior to the "reaction method," as may be termed that shown by the dotted lines outside the truss in Fig. 54. It is infinitely more elegant, and marks the

FIG. 54.

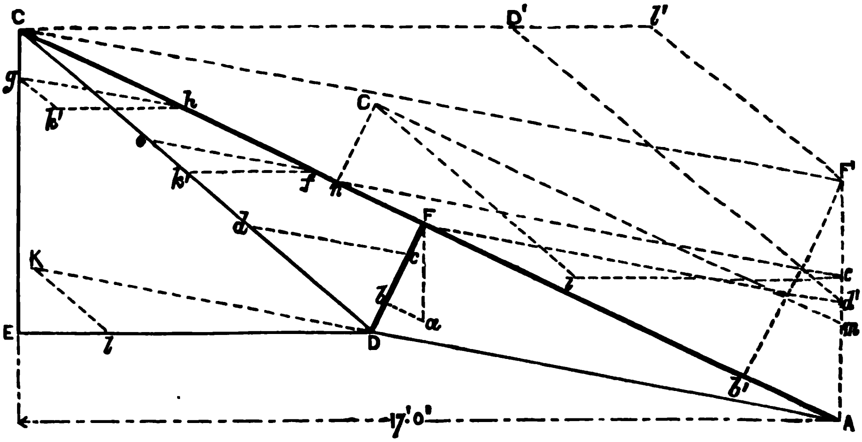


FIG. 55.

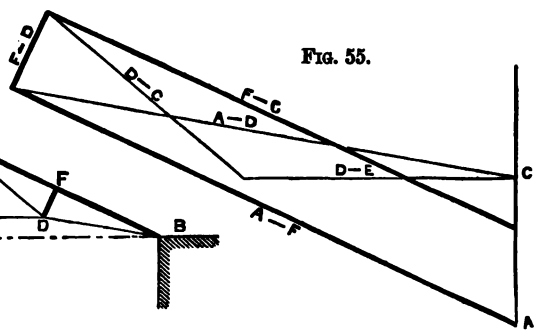
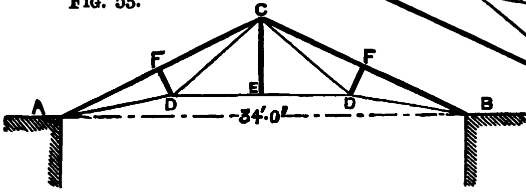


FIG. 53.



subdivisions of the strains better. It is, like the other methods, always used in combination with the elevation of the truss, from which the direction of the different bars has to be obtained. Table XVII. shows the total and separate strains upon the various parts of the truss. The line A C in Fig. 55 represents the total reaction at

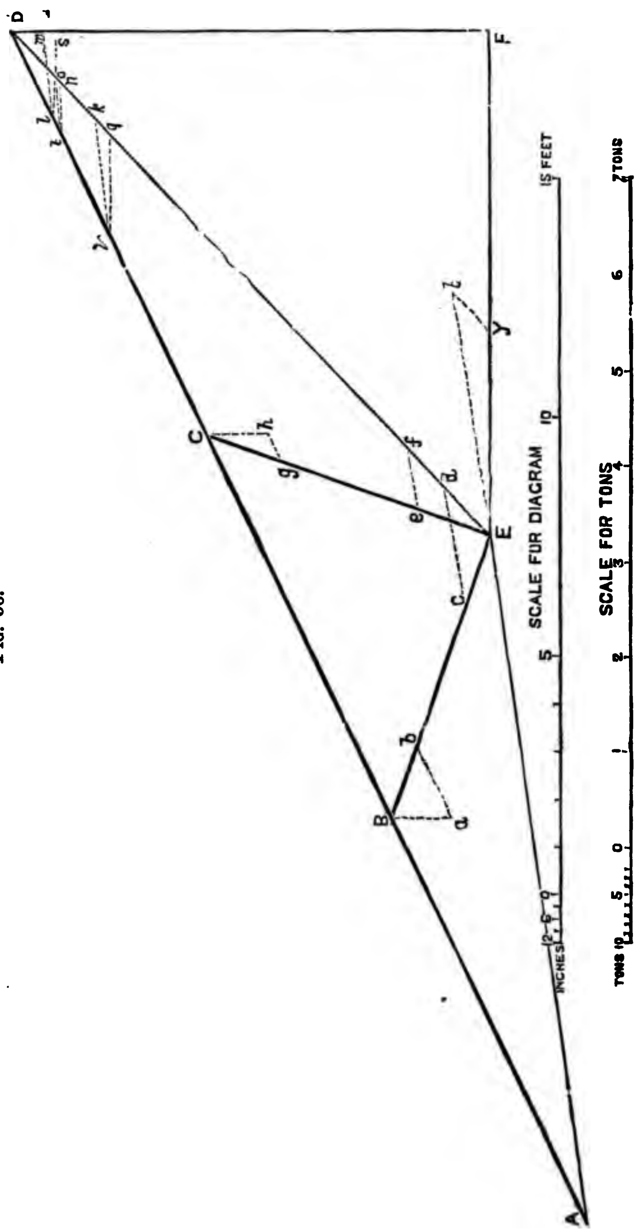
the abutment, and the "polygon of forces" can thus be readily applied to the actual diagram of the roof. Make A c, in Fig. 54, equal 0·75 tons, equal the reaction at A; draw c n parallel to A D to meet the rafter; from the point m, in the line A c, in which c m = the weight supported at A = 0·25 tons; draw m G parallel to the rafter to meet n G, drawn parallel to the strut, and complete the diagram. The junction of the various lines in this diagram will point out the manner in which each bar affects the other, although the relation is not so plainly exhibited as by working out the strains *seriatim* upon the actual truss itself. If the method of ascertaining the strains be worked out by two different diagrams, it will obviate the necessity for checking their accuracy by trigonometrical calculations, although it will be more satisfactory to check the totals by an altogether independent process, than to employ two, which, although varying in detail, depend upon one and the same principle.

CHAPTER XXII.

DOUBLE TRUSS, WITH TWO STRUTS.

When the span of a roof exceeds the limit of about 35 or 40 feet, the simple design illustrated in the last chapter requires to be somewhat modified. It has been before remarked that the introduction of struts in a braced truss or framework, is for the purpose of nullifying the transverse strain, that would otherwise be induced upon the members of the truss. Referring to Fig. 56, it is clear that when the length of the rafter reaches a certain limit, one strut in the centre, as was shown in Fig. 54, would not be sufficient to prevent the two halves of the rafter, into which it divided it, from being affected by transverse strain. Consequently it becomes necessary to use a couple of struts, and divide the whole rafter into three parts. An excellent roof of the form shown in Fig. 56, may be constructed for spans not exceeding 60 feet. When the dimensions are greater, a different and more complicated description of truss must be used, and for very large spans, the circular or segmental form is the best adapted. In tracing the action of the strains in the diagram in Fig. 56, the process will be very analogous to that already described in the previous chapter, making due allowance for the action of the two struts instead of a single one. The distribution of the load will vary, in every instance, according to the number of points at which the rafter is supported, that is, in proportion to the number of struts introduced into the system of trussing.

FIG. 56.

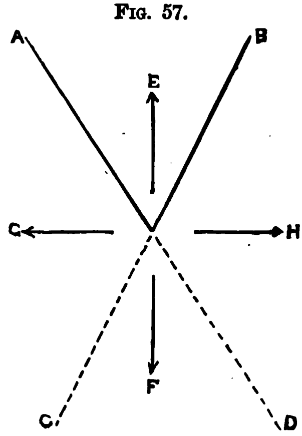

A
B
C
D
E
F
SCALE FOR DIAGRAM
15 FEET
INCHES
SCALE FOR TONS
7 TONS
TONS

A distinction may be here made between the struts and ties, in the manner in which they are affected by the weight at their junction with the principal rafter. In the case in Fig. 56, the portion of the whole load on the half principal situated at the apex B, induces a direct strain upon the part A B of the rafter and upon the strut B E. Similarly, the portion of the load placed at C brings a direct compressive strain upon the parts B C of the rafter, and, by transmission, a strain of the same amount upon A B, and also a direct strain upon C E. But the weight at D produces no direct strain upon the tie D E; that is, the strain upon D E does not start from the same point D as upon the rafter C D, but is transmitted to it at the point E, through the medium of the tie A E. It follows from this that no tie can ever act as a direct support to a weight. While, therefore, a strut may receive its strain either by direct or transmitted weight, a tie can only be strained by the latter description of action. Suspended loads, of course, form an exception to this rule. If either struts or ties are strained by weights acting at any other points than their extremities, the result is a transverse strain, which they are not supposed, theoretically or practically, to be calculated to resist.

Assuming the total load upon the whole principal to equal W, the various subdivisions of it will be arranged as follows:—The weights at B and C will be equal, and double those at A and D. Upon each of the latter there will be $\frac{W}{12}$, and upon each of the former $\frac{W}{6}$. The weight at A is resisted by the vertical reaction of the abutment, and consequently produces no strain upon the truss. The weight at B will strain, either directly or by transmission, every bar in the diagram with the exception of its fellow

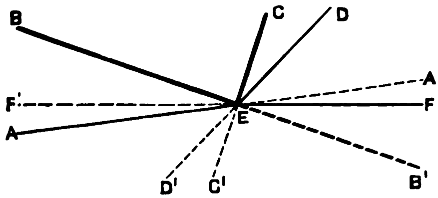
strut C E. If in Fig. 56 the vertical line B a be made equal to $\frac{W}{6}$, and the line $a\ b$ be drawn parallel to the rafter, the succeeding strains may be traced throughout the whole truss, as in the example in the previous chapter. It is sometimes not easy to ascertain exactly, how a strain upon any one bar affects those with which it is connected, or, in other words, whether they are strained compressively, tensilely, or not at all by it. To take the strain along the strut B E on its arrival at the point E, how does it affect C E, A E, E D, and E F? Until it is ascertained how these bars are affected by the thrust along the bar B E, it is impossible to follow out the transmission of the original strain. The problem is to determine which bars are struts and which are ties, when the strain acts upon them in a given direction. In Fig. 57, let A and B be two bars meeting at an apex. If the strain act in the direction of the arrow E, they will both be struts, and if in that of the arrow F, they will both be ties. Should the force be in the direction of the arrow G, the bar A will be a strut, and B a tie, but if in the direction of H, the bar A will be a tie, and B a strut. The rule may be laid down in words as follows:—If the force or strain acts in any direction within the angle formed by the two bars, they are both struts; but if it acts in the direction of, and within, the angle formed by their prolongation, they are both ties. Again, if the

Fig. 57.



strain acts within the angle formed by the original direction of one of the bars, and of the prolongation of the other, then the bar whose prolongation forms one of the sides of the angle is a tie, and the other a strut. To apply this to the truss in the diagram, let the bars in Fig. 58 be represented by the same letters as in Fig. 56, and their prolongations indicated by the dotted lines and corresponding dotted letters.

Fig. 58.

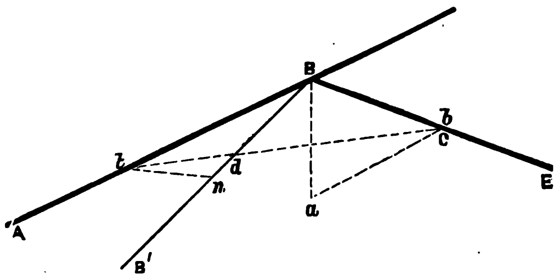


First, let us ascertain how the bars A E and D E are affected by the strain upon the strut B E. Produce B E to B′. The line E B′ represents the direction of the force which lies within the angle A′ E D′, that is, the angle formed by the prolongation of the bars A E and D E. Both these bars are, therefore, ties. If we now take the bars D E and E F, the direction of the strain lies within the angle F E D′, that is, within the angle formed by the direction of one of the bars, and the prolongation of the other, so that D E is in tension and E F in compression. But as E F is not intended to act as a strut, it undergoes no strain from the direct weight at B, as Table XVIII. will show.

In the diagram in Fig. 56, the result of the weight at B is to strain directly the bars A E and D E, and, by transmission, these bars again, and the horizontal tie rod E F. The whole result of the weight at B upon all the separate bars of the truss can be shown by a simple diagram of strains. In Fig. 59, the strains are plotted to double the scale used in Fig. 56, in order to render them clearer, and there will be no difficulty in following their

action, as the letters employed in both diagrams are similar. In the figure, B B′ is drawn parallel to the tie D E, and *b d* and *d t* are the two strains brought upon the tie A E by the weight at B.

FIG. 59.



By making a similar diagram, the strains resulting upon the different members of the truss from the weight at C, can be also ascertained without the necessity of transferring them in succession from one bar to the other. But as the great point for beginners is to trace the action of the strains throughout each successive step, these

TABLE XVIII.

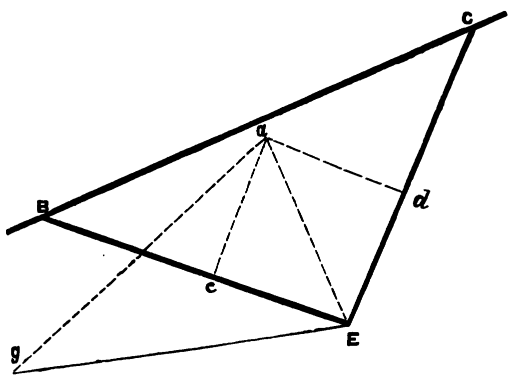
Parts of Truss.	Weight at A	Weight at B	Weight at C	Weight at D	Total Strains.	Remarks.
A B	0	{+0·46, +0·62}	{+0·17, +1·25}	+0·53	+3·03	Rafter.
B C	0	+0·62	{+0·17, +1·25}	+0·53	+2·57	
C D	0	+0·62	+1·25	+0·53	+2·40	
B E	0	+0·43	0·00	0·00	+0·43	Struts.
C E	0	0·00	+0·43	0·00	+0·43	
A E	0	{−0·65, −0·33}	{−0·33, −0·65}	−0·48	−2·44	Ties.
D E	0	{−0·33, −0·07}	{−0·65, −0·14}	−0·10	−1·29	
E F	0	−0·27	−0·55	−0·41	−1·23	Tie rod.

steps are shown in Fig. 56, and the results tabulated in Table XVIII. It is surprising how extremely accurate these calculations can be made, by the aid of a good

diagram. Wherever it is possible, the strains should be checked by an independent mathematical process, but, as in some instances this is not practicable, reliance must be placed upon the accuracy of the diagram and the scale. A mathematical check can generally be applied to some particular parts of the truss, and it should always be made use of when so available.

The method of checking, by mathematical calculation, the accuracy of the results deduced from the diagram, was shown in the last example, and the same process applies in the present case. It may at first appear that the action of the two struts at E is somewhat different to that of only one, and that the strains upon the ties A E and D E, are not equal to one another. But although, in consequence of the difference in the angle made by the two ties with each separate strut, the

Fig. 60.



resulting strains are not individually equal, yet their sum is the same. Instead of dealing with the strains on the struts separately, as in Fig. 56, they can be treated as in Fig. 60. Let B C represent the rafter, and B E, E C the struts. Make E c and E d equal to B b and

C *g*, in Fig. 56, draw *c a* and *d a*, and join *a* E, then *a* E is the resultant of the strains upon the two struts; and if *a g* be drawn parallel to the tie D E, until it meets A E, drawn parallel to the tie A E in *g*, the lines *a g*, E *g* will give the total strains upon the inclined ties A E and D E due to the direct action of the weights at B and C. In the more complicated examples of girder and roof trusses, it is far more convenient to use the resultant of two forces at a given point than to take the forces separately. Sometimes the sum of the forces, as in the present instance, has to be taken into consideration, but in others, it is the difference. It has been shown that the strain upon the horizontal tie rod E F, is that due to only so much of the strain upon A E, as it receives from the rafter, and that it is not influenced by those belonging to the struts. If we take E *l*, in Fig. 56, to represent this amount of strain upon the tie A E, and draw *l y* parallel to D E, then E *y* will represent the strain upon E F, and *l y* the strain upon D E due to the action of the same amount of strain upon A E. In the diagram, E *l* will be found equal to $v\,k + t\,o + l\,m$; E *y* equal to $v\,q + t\,n + l\,s$, and *l y* equal to $q\,k + n\,o + s\,m$. From the Table of strains, the struts are the members of the truss which are the least strained, but practically they are the weakest part of the principal. This is in great measure owing to the large angle between them and the rafter. Although there would not be much difference in the actual theoretical strain upon the strut, if it were placed in the position of the resultant A E, in Fig. 60, yet, practically, the bars placed as in Fig. 56 would have to be made proportionately a great deal stronger. Not only should they have an increased sectional area, but they should be of a form suited to

resist strains of compression. Angle and T iron are the best adapted sections of iron, and round tubes are good forms, but they are not so easy of attachment to the other bars in the truss. The same remark applies to the tie rods. These are frequently made of round iron, which answers well enough in small examples of roofs, but in those of any pretensions to a moderate size span, it will be found preferable to make them of flat bar iron. In the latter case there will be little or no necessity for the welding and forging that is required when round iron is used. A very common, and at the same time, unfounded, apprehension exists with regard to the settlement of roofs, in which the principals are constructed upon the truss system. It is generally supposed that after a short time the tension rods will sag. In order to prevent this, the joints are made with screws or cottars, so that the rods may be tightened up when the sagging takes place, and this method of preventing anticipated sagging has been applied to roofs, with a span of the insignificant dimensions of 25 ft. On the other hand, there are roofs of spans of 50 ft. and more, in which the connections between the bars are simply ordinary bolts, and not the slightest perceptible sagging has ever taken place. There is no doubt that the quality of the workmanship has a great deal to do with the matter, but the work must be very bad, if such expedients have to be resorted to, in roofs of spans less than 80 ft. In a utilitarian point of view, the slight sagging of a roof, provided it be uniform, is not of much consequence, any more than a slight settlement in the foundation of a girder bridge; but it would seriously interfere with the effect of an ornamental ridge in the one case, or a handsome parapet in the other.

CHAPTER XXIII.

DOUBLE TRUSS, WITH THREE STRUTS.

BEFORE proceeding to the examples of curved roofs, one more example of the rectilineal trussed description will be given which will answer for spans of larger dimensions than those already alluded to. It is rather more complicated than its predecessors, but possesses the advantage of having the struts perpendicular to the rafter, and thus as short as possible. The half principal shown in Fig. 61 consists of a primary and two secondary trusses, each of which multiplies the strain, resulting from the weights, on the different members of the truss. We shall only trace the action of one weight, as that of the others is precisely similar, and the mode of analysis is identical with that already fully described. Assuming the value of W, the total load on the half principal, to be one ton, the following will be the distribution on the rafter:—

At A and E there will be a load equal to $\frac{W}{8} = 0 \cdot 125$ tons, and at B, C, and D, a load equal to $\frac{W}{4} = 0 \cdot 25$ tons. In the diagram, make B *a* equal to 0·25 tons, and draw *a b* parallel to the rafter to meet the strut B F, in *b*. The strain upon A B, the lowest part of the rafter, will be found by scaling *a b*, and B *b*, on the same scale, will give the strain upon the strut B F. The strain resulting from the weight at B has now been transferred to the

point F, where three bars of the truss meet, and the question to be determined is, which two of them are affected by the strain acting in the direction of the strut

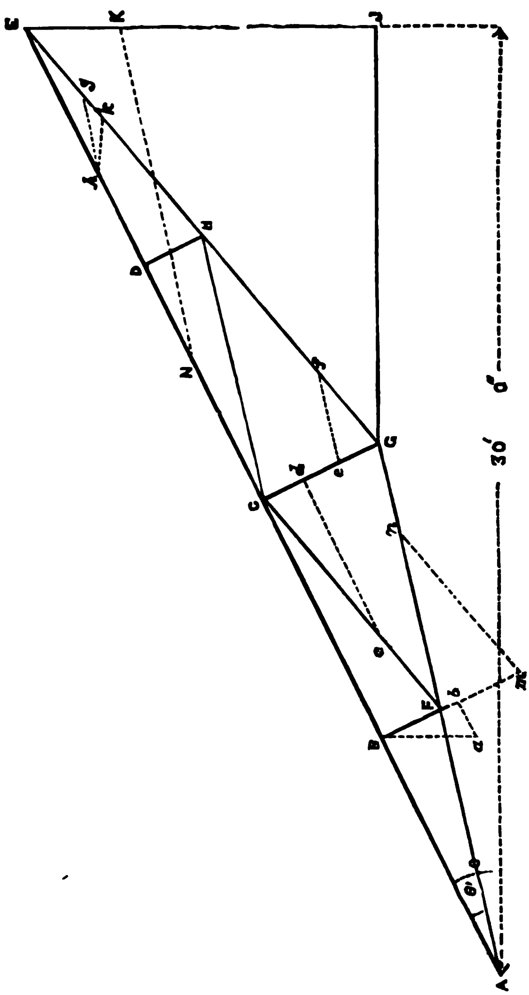


Fig. 61.

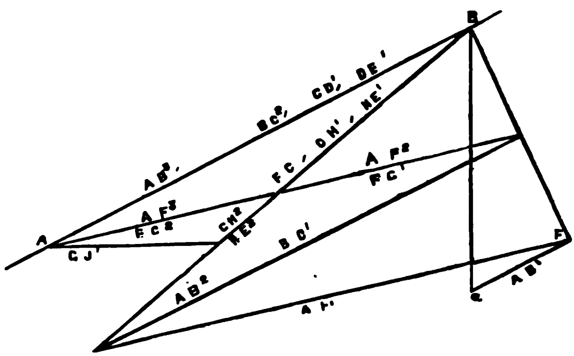
B F. If the rule for determining which are struts and which are ties be referred to, and applied in this case, it will be found that the bars A F, F C, are strained in

tension and F G, in compression, by the strain along B F. But as F G is a tie, and not adapted for resisting compressive strains, the strain along B F must be resolved into its components in the direction of the bars A F, F C. Since the angle A F B is equal to the angle B F C, the components will be also equal to one another, and the strains upon A F and F C will be equal. Make F m = B b, draw $m\ n$ parallel to A F, and either $m\ n$ or F n will give the resulting strain upon the bars A F and F C. The latter of these is transferred to the point C, where it induces compressive strains upon B C and C G, the former of which also strains A B. Make C c equal to F n, equal to $m\ n$; draw $c\ d$ parallel to the rafter, and $c\ d$ and C d will give the resulting strains upon the rafter and strut. The strain upon the strut C G will evidently, when transferred to the point G, pull upon the bars A G, G E. By making e G = C d, and drawing $e\ f$ parallel to A G, the strains upon A F, F G, G H, H E, are at once determined. The pull upon the bar G E is now transferred to the point E, where it induces a strain upon the part E D of the rafter, and subsequently upon every member of the truss except the struts. By taking E g = to G f = $e\ f$, and drawing lines parallel to the different ties, the strains are given by the lines E h, $h\ g$, $h\ k$, and $g\ k$ respectively.

After the explanation that has already been given of this mode of analysis when applied to trusses of different form, it is unnecessary to confuse the diagram in Fig. 61, with the lines that would be required to demonstrate the action of the strain, resulting from the remaining weights at C, D, and E. The process is simply a repetition of the one shown in the figure. In dealing with the weight at D, the pull upon the tie C H must not be forgotten,

and its subsequent effect upon the strut G G and the other members of the half principal. A very good idea of the manner in which any one weight, that at the point B, for example, affects the separate members of the truss, may be obtained by a reference to Fig. 62. The lines

Fig. 62.



and the letters, shown therein, indicate the separate successive strains brought upon the bars. In a similar manner the remaining weights may be thus treated, and their action upon every individual member of the truss ascertained. An inspection of the accompanying Table XIX. will show that, with the exception of the struts and the small secondary ties, the other bars are strained more than once by the same weight.

When the proportions are assigned to the various parts of the truss, as it is manifestly impossible to vary the scantling of the same bar, it must be designed upon the assumption that the bar, throughout its whole length, is subjected to its maximum strain. Although it is highly necessary that the student should be acquainted with the method of determining each separate strain, yet practically, in cases where the truss is of limited dimensions, the maximum strain upon any particular bar is all that is

absolutely needed. Thus, so far as the rafter A E is concerned, though the strains increase from E towards A, the scantling would nevertheless be determined upon the supposition that the rafter was uniformly strained throughout its entire length, by an amount equal to the strain upon A B. If a ready and simple formula could be found for this strain, it might therefore be calculated directly, without any reference to the other parts of the rafter. In Fig. 61, let θ represent the angle of the pitch of the roof,

TABLE XIX.

Parts of Truss.	Weight at					Total Strains.	Remarks.
	A	B	C	D	E		
A B	0	+0·109 +0·464 +0·464	+0·109 +0·929	+0·929 +0·109 +0·464	+0·522	+4·09	
B C	0	+0·464 +0·464	+0·109 +0·929	+0·109 +0·464 +0·929	+0·522	+3·98	Rafter.
C D	0	+0·464	+0·929	+0·109 +0·464 +0·929	+0·522	+3·40	
D E	0	+0·464	+0·929	+0·464 +0·929	+0·522	+3·29	
A F	0	−0·478 −0·238 −0·238	−0·478 −0·478	−0·238 −0·238 −0·478	−0·478	−3·34	
F G	0	−0·238 −0·238	−0·478 −0·478	−0·238 −0·238 −0·478	−0·478	−2·86	
G H	0	−0·238 −0·075	−0·478 −0·157	−0·238 −0·075 −0·157	−0·157	−1·56	Ties.
H E	0	−0·238 −0·075	−0·478 −0·157	−0·478 −0·075 −0·238 −0·157	−0·157	−2·05	
G J	0	0·17	0·35	−0·17 −0·35	−0·35	−1·39	
B F	0	+0·225	0·00	0·00	0·00	+0·225	
C G	0	+0·11	+0·225	+0·11	0·00	+0·445	Struts.
D H	0	0·00	0·00	+0·225	0·00	+0·225	
F C	0	−0·48	0·00	0·00	0·00	−0·48	Ties.
C H	0	0·00	0·00	−0·48	0·00	−0·48	

and θ' the angle between the rafter and the inclined ties. Let S represent the strain upon the strut B F, and R

the total strain upon the end A B of the rafter. Table XIX. shows that there are altogether nine separate strains brought upon A B, so that R may be put equal to $r + r^1 + r^2 + r^3 + \ldots + r^8$. The value of r is obtained at once from the equation $r = \frac{W \times \text{sine } \theta}{4}$. But to find r_1, which is equal in the diagram to the line $c\,d$, we must first find that upon the strut, and the inclined tie F C. Putting S for the strain upon the strut we have $S = \frac{W \times \cos. \theta}{4}$. Let T be put equal to the strain upon the inclined tie F C, and the following proportion obtains; T : S : : sine $(90° - \theta')$ sine $2\,\theta'$, from which we have $T = \frac{S \times \text{sine } (90° - \theta')}{\text{sine } 2\,\theta'}$ Since the sine of an angle equals the cosine of its complement, and sine $2\,\theta'$ equals 2 sine $\theta' \times$ cosine θ', the equation becomes $T = \frac{S \times \text{cosine } \theta'}{2 \text{ sine } \theta' \times \text{cosine } \theta'} = \frac{S}{2 \text{ sine } \theta'}$. Substituting for S its value already arrived at, we have finally $T = \frac{W \times \cos. \theta}{8 \text{ sine } \theta'}$. To find the value of r_1, the equation is $r_1 = T \times \text{cosine } \theta' = \frac{W \times \cos. \theta \times \cot. \theta'}{8}$. It has been shown that the pull on the inclined tie F C, is resolved at the point C, into components—one in the direction of the rafter, equal to r^1, and the other in that of the strut C G. Make the latter equal to S_1, and we have $S_1 = T \times \text{sine } \theta'$, or $S_1 = \frac{W \times \cos. \theta}{8}$. This transferred strain S upon the strut C G, produces a strain upon both the inclined ties A G, G E, and, as the angles they respectively make with the strut are equal, the

resulting strains are also equal. Let T_1 represent the total strain upon A F, or the part of the tie A G, which is strained to a maximum. Then if t, t_1, t_2, &c., be the separate strains, T_1 will be equal to their sum. Similarly, if T_2 represent the corresponding maximum strain upon the tie G E, occurring at H E, it may be taken equal to $p + p_1 + p_2 +$, &c. Let the strain produced upon the inclined ties A G, G E, by the thrust S_1 be made equal to $t = p$. Then $t : S_1 :: \text{sine } (90° - \theta') \text{ sine } 2\,\theta'$, from which we deduce $t = \dfrac{S^1 \times \text{sine } (90° - \theta')}{\text{sine } 2\,\theta'}$. Substituting as before, we have

$$t = \frac{S_1 \times \text{cosine } \theta'}{\text{sine } 2\,\theta'} = \frac{S_1}{2 \text{ sine } \theta'} = \frac{W \times \cos.\, \theta}{16 \text{ sine } \theta'}.$$

But from above, $T = \dfrac{W \times \cos.\, \theta}{8 \text{ sine } \theta'}$; so that $t = \dfrac{T}{2}$. It may be shown that the value of r_2 is identical with that of r_1. In the diagram $r_2 = h$ E, and by proportion $r_2 : t :: \text{sine } (180° - 2\,\theta') : \text{sine } \theta'$, whence

$$r_2 = \frac{t \times \text{sine } 2\,\theta'}{\text{sine } \theta'} = t \times 2 \cos.\, \theta' = \frac{W \times \cos.\, \theta \times \cot.\, \theta'}{8} = r_1.$$

The action of the weight at B has now been accounted for, and we may proceed to consider those at the other apices. The weight at C will obviously affect the rafter and strut in the same manner as that at B; so that, calling the former r_3, and the latter S_2, we obtain $r_3 = r$, and $S_2 = S$. By proportion $r_4 : r_2 :: T : t$; that is, $r_4 = 2\, r_2$. Again, by similar reasoning with respect to the weight at D, $r_5 = r$, $r_6 = r_2$, and $r_7 = r_4 = 2\, r_2$. It only remains now to account for the strain upon the rafter equal to r_8. This is due to the weight at the apex E, and may be obtained from the formula

$$r_8 : \frac{W}{8} :: \text{cosine } (\theta - \theta') : \text{sine } \theta';\ \text{whence } r_8 = \frac{W \times \text{sine } (90° + \theta - \theta')}{8 \text{ sine } \theta'}.$$

The value of R, the maximum strain upon the rafter, may now be summed up. $R = r + r_1 + r_2 +$, &c. $+ r_8$. But from above, $r = r_3 = r_5$; $r_1 = r_2 = r_6$ and $r_4 = r_7 = 2 r_2$. By addition, therefore, $R = 3 r + 7 r_1 + r_8$. Substituting the algebraical value of these respective quantities, the equation becomes

$$R = \frac{3 W \times \text{sine } \theta}{4} + \frac{7 W \times \cos. \theta \times \cot. \theta'}{8} + \frac{W \times \cos. (\theta - \theta')}{8 \text{ sine } \theta'}, \text{ or}$$

$$R = \frac{W}{4}\left\{(3 \text{ sine } \theta) + \frac{(7 \cos. \theta \times \cot. \theta')}{2} + \frac{\cos. (\theta - \theta')}{2 \text{ sine } \theta'}\right\}.$$

By construction angle $\theta = 26°$ and angle $\theta' = 13° 30'$, so that the equation becomes

$$R = 0 \cdot 25 \left\{(3 \times 0 \cdot 43837) + \frac{7(0 \cdot 8987 \times 4 \cdot 174)}{2} + \frac{(0 \cdot 9762)}{2 \times 0 \cdot 2334}\right\}.$$

Reducing and multiplying out, we find $R = 4 \cdot 13$ tons, which agrees practically with the result arrived at by the graphic method of analysis, as indicated in Table XIX. If R_1, R_2, and R_3 represent the strains upon the other parts of the rafter respectively, their values can be readily found from that of R by simple subtraction. Thus, $R_1 = (R - r)$ $R_2 = R - (2 r + r_1)$ $R_3 = R - (3 r + r_1)$.

The other members of the truss may be now considered. The strains upon the struts B F and D H are equal, and given by the formula $S = \frac{W \times \text{cosine } \theta}{4}$. Moreover, since the pull of the inclined tie H C is equal to that of its neighbour F C upon the strut C G, the total strain upon the latter is equal to

$$S + 2 S_1 = \frac{W \times \cos. \theta}{4} + 2\left(\frac{W \times \cos. \theta}{8}\right) \text{ equal to } \frac{W \times \cos. \theta}{2}.$$

It only now remains to find the strains upon A F, H E, and G I, to complete the analysis of the half principal.

As before, put T_1 equal to the strain upon A F equal to $T + t + t_1 + t_2 + A$, and the number of separate strains upon A F will be the same as that upon the end A B of the rafter. As in the case of the rafter, the weight at B causes strains upon A F which are equal to $(T + t + t_1)$. Similarly the weight at C causes a further amount $= (t_2 + t_3)$, while that at D gives rise to others equal to $(t_4 + t_5 + t_6)$, the remaining strain t_7 being caused by the weight at E. But $t = t$, $t_2 = t_3 = T$; $t_4 = t_5 = t$ and $t_6 = T$. Summing up, therefore, the total value of T_1 is given by the equation $T_1 = (4\ T + 4\ t + t_7)$, from which $T_1 = (6\ T + t_7)$ sine $T = 2\ t$. The value of t_7 is obtained from the proportion $t : \frac{W'}{8} :: \cos.\ \theta : \text{sine}\ \theta'$, and $t_7 = \frac{W \times \cos.\ \theta}{8 \times \text{sine}\ \theta'}$. Summing up, $T_1 = \frac{7\ W \cos.\ \theta}{8\ \text{sine}\ \theta'}$, and the strain in tons is equal to $\frac{7 \times 0 \cdot 8987}{8 \times 0 \cdot 2334} = 3 \cdot 36$ tons, which is practically the same result as that given in Table XIX.

In the diagram, Fig. 61, the strain upon F G is equal to $(T_1 - T)$, and can therefore be obtained by simple subtraction. If we make T_2 equal the strain upon H E, it will be equal to $p + p_1 + p_2 +$, &c. But

$$p = t;\ p_1 = \frac{t \times \text{sine}\,(\theta - \theta')}{\text{sine}\,(\theta + \theta')};$$

so that $T_2 = 6\ p + 8\ p_1$. Substituting the values of p and p_1, the equation becomes

$$T_2 = \frac{W \cos.\ \theta}{\text{sine}\ \theta'} \left\{ \tfrac{3}{8} + \frac{\text{sine}\,(\theta - \theta')}{2\ \text{sine}\,(\theta + \theta')} \right\}.$$

The strain upon G H will be equal to that upon H E minus that due to the direct action of the weight at D, and will equal $(T_2 - 2\ p)$. The horizontal tie rod has

now to be considered. Put Q = total strain upon it $= q + q_1 + q_2$, &c.

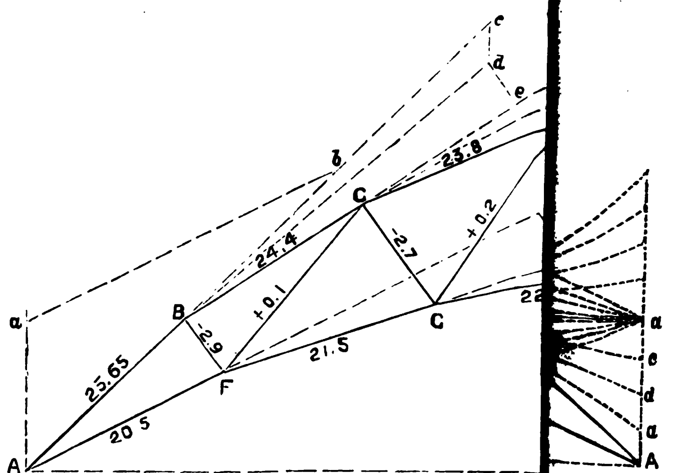
To find q we have the proportion $q : t :: \text{sine } 2\,\theta' : \text{sine } (\theta + \theta')$; when $q = \frac{t \times \text{sine } 2\,\theta'}{\text{sine } (\theta + \theta')}$, but $q_1 = 2\,q$, $q_2 = q_1$, $q_3 = 2\,q$, and $q_4 = 2\,q$; so that the total sum of the strains

$$Q = 8\,q = \frac{8\,t \times \text{sine } 2\,\theta'}{\text{sine } (\theta + \theta')} = \frac{W \times \cos.\,\theta \times \text{sine } 2\,\theta'}{2 \text{ sine } (\theta + \theta')}.$$

Reducing, we obtain finally

$$Q = \frac{W \times \cos.\,\theta \times \cos.\,\theta'}{\text{sine } (\theta + \theta')}.$$

If these formulæ be worked out, the results will be found to agree with those given in the Table XIX. Curved roofs will be next considered, as they constitute the best form for roofs of long span.



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CHAPTER XXIV.

CURVED ROOF TRUSSES.

When the span of a roof exceeds 80′ or 100′ it is more economical to abandon the rectilineal shaped truss, and adopt a form of a semicircular or segmental character. A distinction must here be drawn between the open and the solid type of construction in arched roof principals, as well as in horizontal girders. The plate or solid type is used chiefly in roofs of small spans, examples of which can be seen at the stations of the Metropolitan Railway. In the form of principal represented in Fig. 63, the upper flange is in compression and the lower in tension. The diagonal bars are sometimes in tension, and at others in compression, according to the distribution of the load, the curves selected for the upper and lower flanges, and their inclination to the flanges and one to another. No rule can be laid down with respect to the nature of the strain, that may be induced upon the bars of trusses of this and of a similar character. They must be determined individually by the method already explained. Directly the direction of the forces acting upon the bar, or the direction of their resultant, is known, it can be at once ascertained by simple inspection, whether the resultant strain is one of tension or compression. As a rule, the strains upon the diagonal bars or the web in these trusses are very slight. This arises from the fact, that the greater part of the shearing strain is resisted by

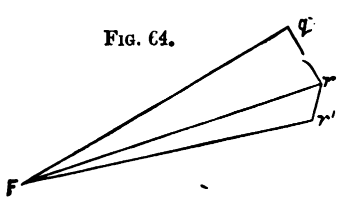
the arched form of the upper flange, instead of the whole of it passing along the bars, as occurs in the open web of the horizontal girder. The amount of the strain upon any two corresponding bars of the upper and lower flanges is not equal, although the inequality diminishes towards the centre of the truss. It may be mentioned that the strains upon the different members of a truss of the form shown in Fig. 63, cannot practically be ascertained by calculation. A diagram must be employed, although calculation affords a means of checking approximately the strains upon the flanges. If the lower flange were horizontal, the truss would be a simple bowstring girder, in which case the strains upon the two flanges would be equal. But the raising of the various members of the lower flange, and the continual alteration of the angle of their inclination, destroys the resemblance.

In the example selected the span of the truss is 120′, and the depth between the upper and lower flanges at the centre 11′ 6″. The whole principal is supposed to be loaded with a uniformly distributed load of 20 tons, which, for all practical purposes, may be considered to be directly supported at the several apices of the upper flange, formed by the junction of the diagonal bars with the flange. It will, therefore, be distributed as follows, over one-half of the truss:—At the apices B, C, D and E, the weight will be 2·22 tons, and half of this, or 1·11 tons, at A. This last weight, being perpendicular to the reaction of the abutment, will produce no strain upon any portion of the truss. The total reaction at the abutment at A will be equal to $4 \times 2 \cdot 22 = 8 \cdot 88$ tons. In the diagram, there are two methods shown of determining the strains, the one proceeding by successive steps, and the other giving the total strains at one ope-

ration. They may be both usefully employed as checks upon each other's accuracy. We will commence with the one on the left-hand side of the diagram. At the point A, there are manifestly three forces making equilibrium, *viz.* the vertical reaction, and the strains upon the bars A B and A F respectively. Since the directions of all these forces are known, and the value of one of them as well, the values of the other two can be readily ascertained. From the point A draw A *a*, vertical, and equal to 8·88 tons. Draw *a b* parallel to the tie A F, to meet A B, produced to *b*. Then A *b* and *a b*, measured on the same scale, will give the strains upon the end bars of the upper and lower flange respectively. Having found the amount of the strains, it remains to determine their character. The reaction at the abutment gives rise to a force acting in the direction of A *a*. If we imagine the bar A F prolonged through A, it is evident that the direction of the force A *a* will be in the angle formed by the original bar A B, and the prolongation of the other bar A F. Consequently the force A *a* will compress the former bar and stretch the latter. At the point B there are four forces to be taken into consideration — the strains upon A B, B C, B F, and the weight at the apex B. Of these four, the direction and amount of two are already known, so that the remainder can be easily determined. Produce B *b* to *c*, making B *c* equal to A *b*, equal to the strain already found for the bar A B. From the point *c* draw *c d* vertically, and equal to the weight at the apex $b = 2 \cdot 2$ tons; join B *d*, which is the resultant of the two known forces at B. If B C be produced to *e* to meet *d e*, drawn parallel to B F, then B *e* and *d e* are the strains upon the bars B C and B F. Their character, whether tensile or compressive,

can be found in the same manner as before. By examining the direction of the resultant B d, with respect to the bars B C and B F, it will be seen that the former is in compression and the latter in tension. At the apex C there are five forces to be accounted for, *viz.* the weight at the apex, and the strains upon the bars B C, C D, F C, C G. But to find that on F C we must refer to the point F. It should be stated here that until we arrive at the diagonal bar H E, the strains upon the preceding bars F C and C D are too small to be made visible on the scale we are obliged to use. But this will not affect the explanation. To resume the analysis, at the point F, there are four forces acting, the strains upon A F and B F, both of which are known, and those upon F C, F G, which are required to be known. First, prolong A F to q, making F q equal to $a\,b$, the strain previously determined for the bar A F, and from q draw $q\,r$ parallel to B F, making $q\,r$ equal to $d\,e$, the amount of the strain upon B F. Join F r, which in reality is the resultant of these two forces, but as the strain upon F C is so small, it practically coincides with the prolongation of the bar F G, and so becomes equal to the strain upon it. A reference to the diagram in Fig. 64, which is not drawn to scale, will explain this fully. If the diagram of strains be drawn to a large scale, the strains upon the bars F C and G B will be rendered manifest, as is that upon the bar H E in Fig. 63. In the diagram in Fig. 64 the bars are lettered to correspond with their representatives in Fig. 63, with the exception that $r\,r^1$ represents the strain upon F C, and F r^1 that upon F G. By

Fig. 64.



repeating the operation at the apex C, which has been performed at B, the lines cf, fg, gh, may be obtained, which gives the strains upon the portions of the truss meeting at that point, and so on for the other apices. At the apex E the strain upon the bar H E is sufficiently large to be appreciable, and the first step in the resolution of the forces at that point, consists in finding the resultant between that strain and the strain upon D E. The strain upon the latter is represented by the line E m, that upon the former by $m\,n$, and the resultant by E n.

Although the strains on the different members of the truss cannot be obtained by calculation, yet those upon the central parts of the upper and lower flange can be ascertained in that manner. In the half truss, the load, which is assumed to be uniformly distributed over it, is the force to be resisted, and the whole system is maintained in equilibrium, partly by the vertical reaction of the abutment, and partly by the strains developed in the flanges of the truss at its centre. From what had been stated, the load may be considered to act at its centre of gravity, with a leverage equal to the difference between the length of the semi-span, and the distance of its centre of gravity from the centre. The leverage of the resistance of either flange, will be equal to the depth of truss at the centre, or, accurately, equal to the distance between the centres of gravity of the upper and lower flanges at that point. Put W = the total load on the principal, L the span, x the distance of the centre of gravity from the centre, D the depth of the truss, and S the strain upon either flange. By the question we have $\frac{W}{2} \times \left(\frac{L}{2} - x\right) = S \times D$, from which

we obtain $S = \frac{W(L - 2x)}{4 \times D}$. The equation for the strain upon the flanges of a horizontal girder, under the same conditions of loading as the truss in Fig. 63, is $S = \frac{W \times L}{8 \times D}$. In the case of a horizontal girder, the centre of gravity of the half load may be assumed to be half-way between the centre and the abutment. This assumption is not strictly correct unless the girder be of uniform section, which it never is, except in very small examples, but it is sufficiently so for all practical purposes. This is tantamount to assuming x equal to $\frac{L}{4}$. If this value for x be inserted in the equation for the truss, its identity with that for the horizontal girder will be at once apparent. As a check upon the successive method adopted on one side of the diagram, it will be useful to employ another, which depends upon the principle of the polygon of forces, which has been explained in a previous chapter.

TABLE XX.

Top Flange.		Bottom Flange.		Web.	
Bars.	Strains.	Bars.	Strains.	Bars.	Strains.
A B	25·63	A F	20·5	B F	−2·9
B C	24·40	F G	21·5	F C	+0·1
C D	23·8	G H	22·5	C G	−2·7
D E	24·0	H J	23·5	C D	+0·1
E E	24·1			D H	−2·7
				H E	+0·8
				E J	−1·8

If the length of the lines on the left hand of the diagram in Fig. 63 be compared with those of the corresponding lines on the right hand, they will be found to agree with a sufficient degree of accuracy. The check is demonstrated by completing the polygon as shown in

the diagram on the right of Fig. 63. If the lines do not "close," there is an error either in the data or in the working out of the process.

The strains given in the annexed table are those due to a load uniformly distributed, but it is also necessary to find those resulting from a load, placed in such a manner, that some of the apices are loaded and not the others. The simplest method of accomplishing this, is to consider the effect of each subdivision of the load separately, and tabulate the results, as has already been done in the case of a horizontal girder. It will then be seen, that the diagonal bars in the web of the truss, are subjected to strains of both description, and that consequently, if we suppose the force of the wind to be acting only on one side of the roof, those bars which under a uniform load might be wholly in tension, would be placed altogether in compression, and *vice versâ.* In testing a roof principal, it is not an uncommon occurrence for the engineer to require that it should be loaded only on one side, in order to ascertain its capabilities of resisting a load ununiformly distributed. Some writers lay great stress upon providing a large margin of strength for wind pressure, but there is more theoretical than practical knowledge displayed in such statements. The wind is far more likely to blow a roof off its supports by getting underneath it, than to blow it down by either a vertical pressure, or one normal to its surface. The precaution of securing roofs which have but little or no shelter, by stout wind-ties, should never be omitted. The arrangement of the bars of the web of circular or curved roof trusses may be varied similarly to that in horizontal girders. Both vertical and diagonal bars are sometimes introduced. Examples of this description of

roof truss on a large scale are erected over the Charing Cross and Cannon Street Railway Stations, and in numerous other similar positions. They form no exception to the rule, that when a greater number of diagonal bars are used than are necessary, the result is a corresponding waste of material.

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